Math 142
Spring, 2005 Name: **Solutions**
Worksheet - §2.8

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1. Find $\left(f^{-1}\right)'(x)$ at $x=1$ if $f(x)=x^3-3x^2+2x+1$.

$$f'(x) = 3x^2 - 6x + 2$$
$$f^{-1}(1) = t \text{ means } f(t) = 1$$

This is true when $t=0$ since
$$f(0) = 0^3 + 3(0)^2 + 2(0) + 1$$

So, we have
$$\left(f^{-1}\right)'(1) = \frac{1}{f'(0)} = \frac{1}{3(0)^2 - 6(0) + 2} = \frac{1}{2}$$

2. Find $\left(f^{-1}\right)'(x)$ at $x=1$ if $f(x)=x^5+3x^3-2x^2+x-2$.

$$f'(x) = 5x^4 + 9x^2 - 4x + 1$$
$$f^{-1}(1) = t \text{ means } f(t) = 1$$

This happens when $t=1$ since
$$f(1) = 1 + 3 - 2 + 1 - 2 = 1$$

So, we have
$$\left(f^{-1}\right)'(1) = \frac{1}{f'(1)} = \frac{1}{5(1)^4 + 9(1)^2 - 4(1) + 1} = \frac{1}{11}$$

3. Find $\left(f^{-1}\right)'(x)$ at $x=3$ if $f(x)=\sqrt{x^2-7}$ for $x \geq \sqrt{7}$.

$$f'(x) = \frac{x}{\sqrt{x^2-7}}$$
$$f^{-1}(3) = t \text{ means } f(t) = 3$$

This happens when $t=4$ since
$$\sqrt{t^2-7} = 3$$
$$t^2 - 7 = 9$$
$$t^2 = 16$$
$$t = 4$$
$$\left(f^{-1}\right)'(3) = \frac{1}{f'(4)} = \frac{1}{\frac{4}{\sqrt{4^2-7}}} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

4. Find $\left(f^{-1}\right)'(x)$ at $x=-9$ if $f(x)=x^5+3x^3-2x^2+x-2$.

$$f'(x)=5x^4+9x^2-4x+1$$
$$f^{-1}(-9)=t \text{ means } f(t)=-9$$

This happens when $t=-1$

$$f(-1)=(-1)^5+3(-1)^3-2(-1)^2-1-2$$
$$=-1-3-2-1-2=-9$$
$$\left(f^{-1}\right)'(-9)=\frac{1}{f'(-1)}$$
$$=\frac{1}{5(-1)^4+9(-1)^2-4(-1)+1}$$
$$=\frac{1}{19}$$

5. Find $\left(f^{-1}\right)'(x)$ at $x=-3$ if $f(x)=1+3x+x^3$.

$$f'(x)=3+3x^2$$
$$f^{-1}(-3)=t \text{ means } f(t)=-3$$

$t=-1$ works, since

$$f(-1)=1+3(-1)+(-1)^3=1+3-1=3$$

So, we have

$$\left(f^{-1}\right)'(-3)=\frac{1}{f'(-1)}=\frac{1}{3+3(-1)^2}=\frac{1}{6}$$