Math 142, Fall 2004 **Chain Rule Worksheet**

Here are some problems to help practice using the Chain Rule

1. $\frac{d}{dx}\left[x^2-2x+3\right]=2x-2$

2. $\frac{d}{dx}\left[(x^2-2x+3)^2\right]=2(x^2-2x+3)(2x-2)$

3. $\frac{d}{dx}\left[\sqrt{x^2-2x+3}\right]=\frac{1}{2}(x^2-2x+3)^{-1/2}(2x-2)=\dfrac{x-1}{\sqrt{x^2-2x+3}}$

4. $\frac{d}{dx}\left[\sin(x^2-2x+3)\right]=\cos(x^2-2x+3)\cdot(2x-2)$

5. $\frac{d}{dx}\left[\sqrt{\sin(x^2-2x+3)}\right]=\frac{1}{2}(\sin(x^2-2x+3))^{-1/2}\cdot\cos(x^2-2x+3)\cdot(2x-2)=\dfrac{(x-1)\cos(x^2-2x+3)}{\sqrt{\sin(x^2-2x+3)}}$

6. $\frac{d}{dx}\left[\dfrac{(x^4+1)^7}{\tan(x^4+1)}\right]=\dfrac{7(x^4+1)^6\tan(x^4+1)-(x^4+1)^7\sec^2(x^4+1)}{\tan^2(x^4+1)}\cdot 4x^3$

7. $\frac{d}{dx}\left[\dfrac{(x^4+1)^7}{\tan(x^{10}+x)}\right]=\dfrac{7(x^4+1)^6\cdot 4x^3\cdot\tan(x^{10}+x)-(x^4+1)^7\sec^2(x^4+1)\cdot(10x^9+1)}{\tan^2(x^{10}+x)}$

8. $\frac{d}{dx}\left[\cos x\sin x\tan x\right]=$

$$\begin{aligned}\frac{d}{dx}\left[\cos x\sin x\tan x\right]&=-\sin x\sin x\tan x+\cos x\cos x\tan x+\cos x\sin x\sec^2 x\\&=-\sin^2 x\tan x+\cos x\sin x+\tan x\\&=\tan x(1-\sin^2 x)+\cos x\sin x\\&=\tan x\cos^2 x+\cos x\sin x\\&=2\cos x\sin x\end{aligned}$$

Why go through all of this? Because, simplifying first,:

$$\frac{d}{dx}\left[\cos x\sin x\tan x\right]=\frac{d}{dx}\left[\cos x\sin x\cdot\frac{\sin x}{\cos x}\right]=\frac{d}{dx}\left[\sin^2 x\right]=2\sin x\cos x$$

9. $\frac{d}{dx}\left[\cos(\sin(\tan x))\right]=-\sin(\sin(\tan x))\cdot\cos(\tan x)\cdot\sec^2 x$

10. $\frac{d}{dx}\left[\tan(\sqrt[4]{x^2+2x-10})\right] = \sec^2\left(\sqrt[4]{x^2+2x-10}\right)\cdot\frac{1}{4}(x^2+2x-10)^{-3/4}\cdot(2x+2)$

11. $\frac{d}{dx}\left[\sqrt[4]{\tan(x^2+2x-10)}\right] = \frac{1}{4}(\tan(x^2+2x-10))^{-3/4}\cdot\sec^2(x^2+2x-10)\cdot(2x+2)$

12. $\frac{d}{dx}\left[\tan^2(\sqrt[4]{x^2+2x-10})\right]$

$$= 2\tan\left(\sqrt[4]{x^2+2x-10}\right)\cdot\sec^2\left(\sqrt[4]{x^2+2x-10}\right)\frac{1}{4}(x^2+2x-10)^{-3/4}\cdot(2x+2)$$