80. A group that has only a finite number of subgroups must be finite.

81. An infinite group is cyclic if and only if it is isomorphic to each of its nontrivial proper subgroups.

82. Let G be a group.

    (a) Suppose $G = \langle a, b \rangle$, that is G is a group containing the elements a and b and all other elements required by the definition of a group. You can think of this as though H is a group which contains the elements a and b and G is the smallest subgroup of H which contains a and b. G is called the *free group on* $\{a, b\}$. What do the elements of G look like?

    (b) Suppose $G = \langle a, b \rangle$ is Abelian. Can you improve upon your answer above for this case? Note: we often describe groups like this using a technique often referred to as "generators and relations". This group would be $\left\langle a, b : aba^{-1}b^{-1} = e \right\rangle$ or $\langle a, b : ab = ba \rangle$.

    (c) Suppose $G = \langle a, b \rangle$ is Abelian with $\circ(a) = n < \infty$ and $\circ(b) = m < \infty$. Can you improve your answer above? Note: using generators and relations, this group is $\left\langle a, b : aba^{-1}b^{-1} = a^n = b^m = e \right\rangle$.

    (d) Suppose $G = \left\langle a, b : aba^{-1}b^{-1} = a^n = b^m = e \right\rangle$. What can you say about $|G|$?

83. In Σ_3 determine

    (a) $\circ((12))$, $\circ((13))$

    (b) $\langle (12), (13) \rangle$

    (c) Does your answer contradict what you saw in problem #82? Explain.

84. Prove every finitely generated Abelian torsion group is finite.

85. Let G be an abelian group with $a, b \in G$ and $\circ(a) = n < \infty$ and $\circ(b) = m < \infty$. Prove there is an element $c \in G$ such that $\circ(c) = \text{lcm}(m, n)$.

86. If G is a finite abelian group in which the number of solutions in G of the equation $x^n = e$ is at most n for every positive integer n. Prove that G must be a cyclic group.