10. Express each of the the following as a product of disjoint cycles:

    (a) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 2 & 1 & 4 & 5 & 6 & 3 & 8 & 9 \end{pmatrix}$

    (b) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 8 & 1 & 7 & 5 & 2 & 6 & 3 & 9 \end{pmatrix}$

11. Verify the following:

    (a) $(12)(23)(34) = (1432)$

    (b) $(1234)(2345) = (13524)$

    (c) $(12)(53214)(23) = (245)$

    (d) $(7236)(85)(571)(1537)(486) = (1348)(27)(56)$

12. Show that the inverse of $(a_1 a_2 \cdots a_k)$ in Σ_n is $(a_1 a_k a_{k-1} \cdots a_3 a_2)$.

13. Let τ be a transposition and let $\sigma \in \Sigma_n$. Prove that $\sigma\tau\sigma^{-1}$ is a transposition.

14. Let σ and τ be disjoint cycles in Σ_n. Prove that $\sigma\tau = \tau\sigma$.

15. (a) Let σ and τ be disjoint cycles in Σ_n with $\sigma^k = \tau^m = (1)$. Prove $(\sigma\tau)^{\mathrm{lcm}(k,m)} = (1)$.

    (b) Find an example which shows that the disjoint condition above is necessary, i.e. find σ, $\tau \in \Sigma_n$ with $\sigma^k = \tau^m = (1)$ but $(\sigma\tau)^{\mathrm{lcm}(k,m)} \neq 1$.