Quotients

97. Let G' be the smallest subgroup of G containing by $\{xyx^{-1}y^{-1} \mid x, y \in G\}$, called the commutator subgroup of G. Show that
    (a) $G' \lhd G$
    (b) G/G' is abelian
    (c) If G/N is abelian then $G' \subset N$
    (d) If $H < G$ and $G' \subset H$ then $H \lhd G$.
98. Let G be the group of real numbers under addition and let N be the subgroup of G consisting of all the integers. Prove that G/N is isomorphic to the group of all complex numbers of absolute value 1 under multiplication.
99. Let G be a group with H a subgroup of finite index. Show that there exists a normal subgroup N of G contained in H and also of finite index.
100. If N is the ideal of all nilpotent elements in a commutative ring R then R/N is a ring with no nonzero nilpotent elements.
101. Let $\phi : R \to S$ be a homomorphism of commutative rings with $\ker \phi = K$, $I \lhd R$ and $J \lhd S$
    (a) $\phi^{-1}(J)$ is an ideal in R that contains K
    (b) If ϕ is surjective, then $\phi(I)$ is an ideal in S.
    (c) If ϕ is not surjective $\phi(I)$ need not be an ideal in S.
    For the following, ϕ is surjective.
    (d) If P is a prime ideal in R that contains K then $\phi(P)$ is a prime ideal in S.
    (e) If Q is a prime ideal in S then $\phi^{-1}(Q)$ is an ideal in R that contains K.
    (f) There is a one-to-one correspondence between the set of all prime ideals in R that contain K and the set of all prime ideals in S given by $P \mapsto \phi(P)$.
    (g) If $I \lhd R$ then every prime ideal in R/I is of the form P/I where $P \lhd R$ is a prime ideal that contains I.
102. If $\text{char}\,(F) = p$, then $\mathbb{Z}_p$ is a subfield of F.