For the following, let Q_8 be the quaternion group, and let $\mathbb{H}$ be the ring of real quaternions.

36. Let G be the subgroup of $GL_2(\mathbb{C})$ generated by $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$.
    (a) Show that G is a non-abelian group of order 8.
    (b) Show that G is isomorphic to the quaternion group.

37. Let G be the subgroup of $GL_2(\mathbb{C})$ generated by $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.
    (a) Show that G is a non-abelian group of order 8.
    (b) Show that G is not isomorphic to the quaternion group.

38. Determine all subgroups of Q_8.

39. If G is a group then $C = \{a \in G \mid ax = xa \ \forall \ x \in G\}$ is called the center of G.
    (a) Show C is an abelian subgroup of G.
    (b) What is the center of Q_8?

40. Let $q \in \mathbb{H}$. Prove that $qi = iq \Leftrightarrow q \in \mathbb{C}$.

41. Determine the center of $\mathbb{H}$.

42. Determine conditions on $a, b, c, d \in \mathbb{R}$ so that $q^2 = -1$ where $q = a + bi + cj + dk$.

43. Show that the equation $x^2 = -1$ has infinitely many solutions in $\mathbb{H}$.

44. Prove that $\mathbb{H}$ is a division ring.