

IMMERSE Algebra Homework - Summer 2005  
The Paper Revisited 2

- Reread section 2 of the paper, including the following:
- Reread Theorem 2.2. We will talk about many of these topics on Friday and Tuesday. Remember that he really meant

$$K[H] \cong \prod_{d|n} K(\xi_d)^{a_d}$$

- Be prepared to explain / discuss the proof of Proposition 2.3, including any homework problems you use to explain the proof. Notice the typo fixed in class.

**Proposition 2.3.** For all rings  $R_1, R_2$  and  $R$  and a group  $G$ :

- (a) The group ring  $(R_1 \times R_2)[G]$  is reversible (symmetric) if and only if both  $R_1[G]$  and  $R_2[G]$  are reversible (symmetric).
- (b) The group ring  $R[G]$  over an abelian **torsion** group  $G$  is reversible (symmetric) if and only if  $R[H]$  is reversible (symmetric) for all finite subgroups  $H$  of  $G$ .

- Look at the comment on the bottom of page 283. Notice that you have checked most of these calculations already.
- Be prepared to explain / discuss the proof of Proposition 2.4, including any homework problems you use to explain the proof.

**Proposition 2.4.** With the notation above (well, in the paper), for all  $a, a' \in K[Q_8]$ , the equality  $aa' = a'a$  holds iff the rank of the matrix  $\begin{pmatrix} d_u & d_v & d_w \\ d'_u & d'_v & d'_w \end{pmatrix}$  is less than 2.