

IMMERSE Homework - Algebra

Summer 2005

1. Quickly *skim* the paper looking for key topics and terms. For each classify the term in the following table as:

- (a) A topic or term that I know something about
- (b) A topic or term that I've heard of, or
- (c) A topic or term I've never heard of before

For each of the topics in the "Know Something About" column, give a precise definition of the term.

Know something about	Heard of	Unknown

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The following problem is due at the beginning of the class on Tuesday, 28 June. It will be used to help us ascertain which of the following topics are familiar and which we may spend more time on, so please be honest. You are not expected to know or recognize all of the terms list.

2. Look at the following list of words.

- | | | | |
|------------------------|--------------------|---------------------------|--------------------------|
| group | subgroup | abelian group | normal subgroup |
| order of an element | order of a group | direct product of group | direct sum of group |
| symmetric group | permutation | cyclic group | quaternion group |
| torsion group | torsion-free group | ring | subring |
| ideal | center | idempotent | characteristic of a ring |
| identity of a ring | field | characteristic of a field | extension field |
| degree of an extension | cyclotomic field | root of unity | primitive root of unity |
| \mathbb{Z}_n | group ring | | |

For each classify the term as:

- (a) A term that I know something about
- (b) A term that I've heard of, or
- (c) A term I've never heard of before

Know something about	Heard of	Unknown

3. Prove that every commutative ring is symmetric.
4. Prove that every symmetric ring is reversible.
5. A ring with identity which has no non-zero zero divisors is reversible and symmetric.
6. Suppose that R is a subring of S . Prove that if S is reversible then R is reversible.
7. Suppose that R is a subring of S . Prove that if S is symmetric then R is symmetric.

Note: Homework problems 6 and 7 would frequently be written as one in the following way:

Suppose that R is a subring of S . Prove that if S is reversible (symmetric) then R is reversible (symmetric).

8. If R is a reversible ring and $a, b, c \in R$ such that $abc = 0$, what other products of three elements must be zero? Form a conjecture and prove your claim.
9. If R is a symmetric ring and $a, b, c \in R$ such that $abc = 0$, what other products of three elements must be zero? Form a conjecture and prove your claim.