54. (a) If $H \cong G$ as groups, show that $R[G] \cong R[H]$ as rings.
    (b) Find an example to show that the converse is not necessarily true.

55. Let G be a group with $g \in G$ and $g^m = e$. Let R be a commutative ring with identity 1. Show $(1-g)\left(1+g+\cdots+g^{m-1}\right) = 0$ in $R[G]$. (note: $1-g$ is shorthand for the element $1 \cdot e + (-1) \cdot g$ of $R[G]$)

56. Let $G = \{g_1, g_2, \ldots, g_n\}$ be a finite group. Prove $N = g_1 + g_2 + g_3 + \cdots + g_n$ is in the center of $R[G]$.

57. Prove or disprove: if $| G | < \infty$ then $R[G]$ has zero divisors. (In this case, what happens if R is not commutative and / or R does not have unity? What happens if $G = \langle e \rangle$?)

58. Let R be a commutative ring with identity and let G be a finite group with $|G| > 1$. Show that $R[G]$ contains zero-divisors.

59. Show that $\mathbb{R}[Q_8]$ is not a division ring.

    The following problems are from later in the paper, so you should just find the definitions required for the notation. We will return to other ideas used in these proofs later this summer.

60. Verify (some) the equations in (S) on the fifth page (p. 284) of the paper. Namely, with notation as in the paper, verify:

$$tt' + u \cdot u' + v \cdot v' + w \cdot w' = b_1$$

$$t \cdot t' + uu' + vv' + ww' = b_{-1}$$

$$tu' + ut' + vw' + w \cdot b' = b_i$$

$$t \cdot u' + u \cdot t' + v \cdot w' + wv = b_{-i}$$

$$tb' + uw' + vt' + wu' = b_j$$

$$t \cdot v' + uw' + v \cdot t' + w \cdot u' = b_{-j}$$

$$tw' + uv' + v \cdot u' + wt' = b_k$$

$$t \cdot w' + u \cdot v' + vu' + w \cdot t = b_{-k}$$

61. Verify $aa' = 0$ while $a'a \neq 0$ for a and a' as defined in the first displayed equations on the seventh page of the paper (page 286).