45. Prove that $R[x]$ is a commutative ring with identity whenever R is.

46. If R is an integral domain, prove that for $f(x), g(x) \in R[x]$,

$$deg(f(x)g(x)) = deg(f(x)) + deg(g(x))$$

47. If R is an integral domain with identity, prove that any unit in $R[x]$ must already be a unit in R.

48. Let R be a commutative ring with no nonzero nilpotent elements. If $f(x) = (a_0, a_1, \ldots, a_m, 0, \ldots) \in R[x]$ is a zero divisor, prove there is a $b \neq 0$ in R so that $ba_0 = ba_1 = \cdots = ba_m = 0$.

    For the following, let R be a commutative ring with identity.

49. If R is a subring of S, then $R[G]$ is a subring of $S[G]$ for any group G.

50. Let $H < G$. Show that $R[H]$ is a subring of $R[G]$.

51. Show that the set of all elements of $R[G]$ whose coefficients sum to zero is a subring of $R[G]$.

52. In $\mathbb{Z}[\Sigma_3]$ let

$$\alpha = 3(12) - 5(23) + 14(123)$$
$$\beta = 6(1) + 2(23) - 7(123)$$

    Compute the following:

    (a) $\alpha + \beta$
    (b) $2\alpha - 3\beta$
    (c) $\alpha\beta$
    (d) $\beta\alpha$
    (e) α^2

53. Explain why $\mathbb{R}[Q_8]$ is not the same ring as the ring of quaternions.