131. Let K be a field of characteristic $p \neq 2$. Show $\Phi_4(x) = x^2 + 1$

132. Let F be a field and let a and b be elements of some extension field of F.

   (a) Give the definition of $F(a,b)$.
   (b) Give the definition of $F(a)(b)$. (note: $F(a)(b)$ is interpreted as $(F(a))(b)$)
   (c) Use these definitions to explain why $F(a,b) = F(a)(b)$.

133. Suppose E is an extension field of the field F. Prove $E = F$ if and only if $[E : F] = 1$.

134. Suppose E is an extension field of the field F and K is an extension field of the field K (so, $F \subseteq E \subseteq K$). Prove $[K : F] = [K : E][E : F]$.

135. Let E be an extension field of the field F. Suppose $[E : F]$ is prime. Suppose $\mathcal{F}$ is a field with $F \subseteq \mathcal{F} \subseteq E$. Prove $F = \mathcal{F}$ or $E = \mathcal{F}$.

136. Let $f(x) = x^3 + x^2 + \overline{1} \in \mathbb{Z}_2[x]$. Suppose a is a zero of $f(x)$ in some extension of $\mathbb{Z}_2$. Then $\mathbb{Z}_2(a) \approx \mathbb{Z}_2[x] / \langle f(x) \rangle$. Write each of the following in the form $c_0 + c_1 a + c_2 a^2$ where $c_0, c_1, c_2 \in \mathbb{Z}_2$.

   (a) a^3
   (b) a^{-1}
   (c) a^{50}

137. Prove:

   (a) $\mathbb{Q}\left(\sqrt{2} + \sqrt{3}\right) = \mathbb{Q}\left(\sqrt{5 + \sqrt{24}}\right)$
   (b) $\mathbb{R}(a + bi) = \mathbb{C}$ where $a + bi \in \mathbb{C}$ with $a, b \in \mathbb{R}$ and $b \neq 0$.
   (c) $\mathbb{Q}(i) \neq \mathbb{C}$
   (d) $\mathbb{Q}\left(i + \sqrt{2}\right) = \mathbb{Q}\left(\sqrt{i}\right)$
   (e) Is $\mathbb{R}[i] = \mathbb{C}$?

138. Compute $\left[\mathbb{Q}\left(\sqrt{2}\right) : \mathbb{Q}\right]$ and $\left[\mathbb{Q}\left(\sqrt{3}\right) : \mathbb{Q}\right]$ and show $\mathbb{Q}\left(\sqrt{2}\right) \neq \mathbb{Q}\left(\sqrt{3}\right)$.