103. Let G_1 and G_2 be groups. Suppose $H_1 \triangleleft G_1$ and $H_2 \triangleleft G_2$. Let $G = G_1 \oplus G_2$ and let $H = H_1 \oplus H_2$. Prove that $H \triangleleft G$.

104. Suppose H and K are subgroups of a group G and $H \cap K = \{e\}$. Prove that every element of $HK = \{hk : h \in H,\ k \in K\}$ can be written uniquely in the form hk where $h \in H$ and $k \in K$.

105. Suppose H and K are subgroups of a group G and $H \cap K = \{e\}$ and $G = HK$. Prove $G \approx H \times K$.

106. Let $G \neq \{e\}$ be a group. Show that $\{(g, g) \mid g \in G\}$ is a subgroup of $G \times G$ which is not of the form $H \times K$ with $H, K < G$.

107. Let R, S be rings with identity. Let I be an ideal in $R \times S$. Define

$$I_R = \{r \in R \mid (r, y) \in I \text{ for some } y \in S\}$$

$$I_S = \{s \in S \mid (x, s) \in I \text{ for some } x \in R\}$$

   (a) Show $I = I_R \times I_S$.

   (b) Show I_R is an ideal in R and I_S is an ideal in S.
   Notice: this allows us to conclude that every ideal in $R \times S$ is of the form $H \times K$ where H is an ideal in R and K is an ideal in S.

108. Suppose I and J are ideals of a ring R and $I \cap J = \{0\}$ and $R = I + J$, then $R \approx I \oplus J$.

109. Show $K[G \times H] \approx K[G][H]$ for every field K, and groups G and H.

110. Show $(R_1 \times R_2)[G] \approx R_1[G] \times R_2[G]$ where R_1 and R_2 are commutative rings and G is a group.

111. Show that $G = Q_8 \times \mathbb{Z}$ is not hamiltonian.