Quiz 11 Name: Section:

1. $f(x,y) = 2x^3 - 4xy + 6y^3$, find f_y, f_{yy}, and f_{yx}.

$f_y = -4x + 18y^2$

$f_{yy} = 36y$

$f_{yx} = -4$

2. $f(x,y) = 2x^3 - 4xy + 6y^3$, find f_x, f_{xx}, and f_{xy}.

$f_x = 6x^2 - 4y$

$f_{xx} = 12x$

$f_{xy} = -4$

3. $f(x,y) = \sqrt{4 - x^2 + 8y}$, find f_y and f_x.

$f_y = \frac{1}{2}(4-x^2+8y)^{-1/2} \cdot 8 = \frac{4}{\sqrt{4-x^2+8y}}$

$f_x = \frac{1}{2}(4-x^2+8y)^{-1/2} \cdot (-2x) = \frac{-x}{\sqrt{4-x^2+8y}}$

4. Classify the critical points when $f(x,y) = 4x^4 - 8x^2 - 9y^2 + 6y + 3$.

$f_x = 16x^3 - 16x = 16x(x^2-1)$

$f_x = 0 \qquad 16x(x^2-1) = 0$

$x = 0, \quad x = \pm 1$

$f_y = -18y + 6$

$f_y = 0 \qquad -18y + 6 = 0 \qquad y = \frac{-6}{-18} = \frac{1}{3}$

$f(0, \frac{1}{3}) = 4, \quad f(1, \frac{1}{3}) = 0, \quad f(-1, \frac{1}{3}) = 0$

Critical points

$(0, \frac{1}{3}, 4), \quad (1, \frac{1}{3}, 0), \quad (-1, \frac{1}{3}, 0)$

Use Second Derivative Test

$f_{xx} = 48x^2 - 16$

$f_{yy} = -18$

$f_{xy} = 0$

$D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - [f_{xy}(a,b)]^2$

$D(0, \frac{1}{3}) = (-16)(-18) - 0 = 288 > 0$

and $f_{xx}(0, \frac{1}{3}) = -16 < 0$

so $(0, \frac{1}{3}, 4)$ is a local maximum

$D(1, \frac{1}{3}) = (32)(-18) - 0 < 0$

so ~~(0,~~ $(1, \frac{1}{3}, 0)$ is a saddle point

$D(-1, \frac{1}{3}) = (32)(-18) - 0 < 0$

so $(-1, \frac{1}{3}, 0)$ is a saddle point.