

Spring 2004

Math 308

May 11, 2004

## Final Exam

Name (last, first) \_\_\_\_\_

Signature \_\_\_\_\_

Section# \_\_\_\_\_

Seat# \_\_\_\_\_

### Instructions

1. **Calculators may not be used during the entire exam.**
2. Mark the correct choice on your Scantron form 882-ES using a No. 2 pencil. For your record, also mark your choices on the exam itself. There are 17 questions in this exam. Your grade will depend on your best 15 answers. Each correct answer is worth 4 points, up to a maximum of 60 points.

For official use only! Final Score \_\_\_\_\_

Each problem is worth 4 points. Only your best 15 answers will be considered. Calculators are not allowed.

1. The solution of the differential equation  $\frac{dy}{dx} - (\tan x)y = \sec x$ , with initial condition  $y(0) = -1$  is
  - (a)  $y(x) = (x - 1) \sec x$
  - (b)  $y(x) = (2x - 1) \sec x$
  - (c)  $y(x) = \sin x - \cos x$
  - (d)  $y(x) = 2 \sin x - \cos x$
  - (e)  $y(x) = x \sin x - \cos x$
  
2. An integrating factor for the differential equation  $y' - \frac{y}{x} = x$  is given by
  - (a)  $x$
  - (b)  $1/x$
  - (c)  $\ln x$
  - (d)  $x^2$
  - (e)  $1/x^2$
  
3. Assume that  $y(t)$  is a solution of the equation  $y'' + 7y' + 6y = 0$ . What can you say about  $\lim_{t \rightarrow \infty} y(t)$ ?
  - (a) The limit does not exist.
  - (b) The limit exists only if  $y(0) > 0$ .
  - (c) The limit is always zero, no matter what the initial condition is.
  - (d) The limit exists only if  $y'(0) > 0$ .
  - (e) It is not possible to decide the value of the limit.

4. The general solution of the equation  $y'' + 6y' + 10y = 0$  is

(a)  $y(t) = C_1 e^{3t} \cos t + C_2 e^{3t} \sin t$

(b)  $y(t) = C_1 e^{-3t} \cos 2t + C_2 e^{-3t} \sin 2t$

(c)  $y(t) = C_1 e^{-t} \cos t + C_2 e^{-t} \sin t$

(d)  $y(t) = C_1 e^{-3t} \cos t + C_2 e^{-3t} \sin t$

(e)  $y(t) = C_1 e^{-t} \cos 3t + C_2 e^{-t} \sin 3t$

5. A fifth order equation, has the auxiliary equation  $(r - 1)^3(r^2 + 1) = 0$ . The general solution of the equation is

(a)  $y(t) = C_1 e^{-t} + C_2 t e^{-t} + C_3 t^2 e^{-t} + C_4 \cos t + C_5 \sin t$

(b)  $y(t) = C_1 e^t + C_2 t e^t + C_3 t^2 e^t + C_4 t^3 e^t + C_5 e^{-t}$

(c)  $y(t) = C_1 e^t + C_2 t e^t + C_3 t^2 e^t + C_4 \cos t + C_5 \sin t$

(d)  $y(t) = C_1 e^{-t} + C_2 t e^{-t} + C_3 t^2 e^{-t} + C_4 t^3 e^{-t} + C_5 e^t$

(e)  $y(t) = C_1 e^t + C_2 e^{t^2} + C_3 e^{t^3} + C_4 \cos t + C_5 \sin t$

6. When one tries to find a particular solution of the equation  $y'' + 4y' + 4y = -6t^3 e^{-2t}$  using the method of undetermined coefficients, one should try to find  $y_p(t)$  of the form

(a)  $y_p(t) = A_0 t^3 e^{-2t}$

(b)  $y_p(t) = t(A_0 + A_1 t + A_2 t^2 + A_3 t^3) e^{-2t}$

(c)  $y_p(t) = t^2(A_0 + A_1 t + A_2 t^2 + A_3 t^3) e^{-2t}$

(d)  $y_p(t) = (A_0 + A_1 t + A_2 t^2 + A_3 t^3) e^{-2t}$

(e)  $y_p(t) = A_0 t^6 e^{-2t}$

7. The function  $y_p(t) = t^2 + 1$  is a particular solution of the equation  $y'' - 4y' + 5y = 5t^2 - 8t + 7$ . The solution of the above equation that satisfies the initial condition  $y(0) = 0, y'(0) = 0$  is:

(a)  $y(t) = 2e^{2t} \sin t - e^{2t} \cos t + 10t^2 - 12t + 12$

(b)  $y(t) = 22e^{2t} \sin t - 7e^{2t} \cos t + t^2 + 1$

(c)  $y(t) = 2e^{2t} \sin t - e^{2t} \cos t + t^2 + 1$

(d)  $y(t) = 22e^{2t} \sin t - 7e^{2t} \cos t + 10t^2 - 12t + 12$

(e)  $y(t) = 3e^{2t} \sin t - e^{2t} \cos t + t^2 + 1$

8. The Laplace transform  $Y(s)$  of the solution of the equation  $y'' - 7y' + 6y = t$ , that satisfies the initial condition  $y(0) = 0, y'(0) = 1$  is

(a)  $Y(s) = \frac{s + 1}{s(s^2 - 7s + 6)}$

(b)  $Y(s) = \frac{1 - 6s^3}{s^2(s^2 - 7s + 6)}$

(c)  $Y(s) = \frac{s^2 + 1}{s^2(s^2 - 7s + 6)}$

(d)  $Y(s) = \frac{1 - 6s^2}{s(s^2 - 7s + 6)}$

(e)  $Y(s) = \frac{1}{s^2(s^2 - 7s + 6)}$

9. The inverse Laplace transform of the function  $Y(s) = \frac{1}{(s - 2)(s - 3)}$  is

(a)  $y(t) = e^{3t} - e^{2t}$

(b)  $y(t) = e^{2t} - e^{3t}$

(c)  $y(t) = e^{3t} + e^{2t}$

(d)  $y(t) = 2e^{3t} - 3e^{2t}$

(e)  $y(t) = 3e^{2t} - 2e^{3t}$

10. The vectors  $v_1 = (3, 2)$  and  $v_2 = (4, 3)$  are eigenvectors of the matrix

$$A = \begin{bmatrix} -7 & 12 \\ -6 & 10 \end{bmatrix}$$

What is the general solution to the system  $x' = Ax$ ?

(a)  $C_1v_1 + C_2v_2$

(b)  $C_1e^tv_1 + C_2e^{2t}v_2$

(c)  $C_1e^{2t}v_1 + C_2e^tv_2$

(d)  $C_1e^{2t}v_1 + C_2e^{-t}v_2$

(e)  $C_1e^{-2t}v_1 + C_2e^{2t}v_2$

11. If you were asked to approximate the value of  $y(1)$  of the solution of the problem  $y' = \cos y^2$ ,  $y(0) = 0$ , using Euler's method with  $h = 0.1$ , the formula for the terms in the approximation would be:

- (a)  $t_0 = 0, y_0 = 0, t_{k+1} = t_k + h, y_{k+1} = y_k + h \cos y_k^2$
- (b)  $t_0 = 0, y_0 = 0, t_{k+1} = t_k + h, y_{k+1} = 0.1(y_k + h \cos y_k^2)$
- (c)  $t_0 = 0, y_0 = 1, t_{k+1} = t_k + h, y_{k+1} = y_k + h \cos y_k^2$
- (d)  $t_0 = 0, y_0 = 1, t_{k+1} = t_k + h, y_{k+1} = h(y_k + h \cos y_k^2)$
- (e)  $t_0 = 0, y_0 = \cos 1, t_{k+1} = t_k + h, y_{k+1} = y_k + h \cos y_k^2$

12. A system  $x' = Ax$  has a triple eigenvalue  $r$  but only one eigenvector  $v$  for that eigenvalue. Another linearly independent solution of the system can be found in the form

- (a)  $x(t) = te^{rt}v$
- (b)  $x(t) = te^{rt}v + e^{rt}w$
- (c)  $x(t) = te^{At}v$
- (d)  $x(t) = e^{At}v$
- (e)  $x(t) = e^{rt}v + te^{rt}w$

13. The general solution of the system  $x' = Ax$ , where  $A$  is given by

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$$

is

- (a)  $C_1 e^t \begin{bmatrix} \cos 2t \\ \sin 2t + \cos 2t \end{bmatrix} + C_2 e^t \begin{bmatrix} \sin 2t \\ \sin 2t - \cos 2t \end{bmatrix}$
- (b)  $C_1 e^{-t} \begin{bmatrix} \cos 2t \\ \sin 2t + \cos 2t \end{bmatrix} + C_2 e^{-t} \begin{bmatrix} \sin 2t \\ \sin 2t - \cos 2t \end{bmatrix}$
- (c)  $C_1 \begin{bmatrix} \cos t \\ \sin 2t + \cos 2t \end{bmatrix} + C_2 \begin{bmatrix} \sin 2t \\ \sin 2t - \cos 2t \end{bmatrix}$
- (d)  $C_1 e^t \begin{bmatrix} \cos 2t \\ \sin 2t + \cos 2t \end{bmatrix} + C_2 \begin{bmatrix} \sin 2t \\ \sin 2t - \cos 2t \end{bmatrix}$
- (e)  $C_1 \begin{bmatrix} \cos 2t \\ \sin 2t + \cos 2t \end{bmatrix} + C_2 e^{-t} \begin{bmatrix} 2 \sin 2t \\ \sin 2t - \cos 2t \end{bmatrix}$

14. The particular solution  $y_p$  of the equation  $y'' + 4y = f(t)$  that satisfies  $y_p(0) = y_p'(0) = 0$  is given by

(a)  $\int_0^t f(s) \sin(2s - 2t) ds$

(b)  $\int_0^t f(s) \sin(2t - 2s) ds$

(c)  $\int_0^t f(s) \cos(2s - 2t) ds$

(d)  $\int_0^t f(s) \cos(2t - 2s) ds$

(e)  $\int_0^t f(s) \sin(2s + 2t) ds$

15. The general solution of the equation

$$(2x + \frac{y}{1 + x^2y^2}) dx + (\frac{x}{1 + x^2y^2} - 2y) dy$$

is:

(a)  $x^2 + y^2 + \arctan xy = C$

(b)  $x^2 - y^2 - \arctan xy = C$

(c)  $x^2 - y^2 + \arctan xy = C$

(d)  $y^2 - x^2 + \arctan xy = C$

(e)  $y^2 + x^2 - \arctan xy = C$

16. The general solution of the equation  $y'' + 10y' + 25y = 0$  is given by

(a)  $y(t) = C_1e^{5t} + C_2te^{5t}$

(b)  $y(t) = C_1e^{-5t} + C_2te^{5t}$

(c)  $y(t) = C_1e^{-5t}$

(d)  $y(t) = C_1e^{-5t} + C_2e^{5t}$

(e)  $y(t) = C_1e^{-5t} + C_2te^{-5t}$

17. A particular solution of the equation  $y'' + 16y = 32t^2 - 12$  is

(a)  $y(t) = \cos 4t + 2t^2 - 1$

(b)  $y(t) = \cos 4t + t^2 - 1$

(c)  $y(t) = \sin 4t + 2t^2 + 1$

(d)  $y(t) = \cos 4t + 2t^2 + 1$

(e)  $y(t) = \sin 4t + t^2 - 1$