

Method of Undetermined Coefficients

To find a particular solution to the differential equation

$$ay'' + by' + cy = P_m(t)e^{rt},$$

where $P_m(t)$ is a polynomial of degree m , use the form

$$y_p(t) = t^s(a_mt^m + \cdots + a_1t + a_0)e^{rt};$$

- if r is **not a root** of the auxiliary equation, take $s = 0$,
- if r is a **simple root** of the auxiliary equation, take $s = 1$,
- if r is a **double root** of the auxiliary equation, take $s = 2$.

To find a particular solution to the differential equation

$$ay'' + by' + cy = P_m(t)e^{\alpha t} \cos(\beta t) + Q_n(t)e^{\alpha t} \sin(\beta t),$$

where $P_m(t)$ is a polynomial of degree m and $Q_n(t)$ is a polynomial of degree n , use the form

$$y_p(t) = t^s(a_k t^k + \cdots + a_1 t + a_0)e^{\alpha t} \cos(\beta t) + t^s(b_k t^k + \cdots + b_1 t + b_0)e^{\alpha t} \sin(\beta t),$$

where k is the larger of m and n .

- If $\alpha + \beta i$ is **not a root** of the auxiliary equation, take $s = 0$;
- if $\alpha + \beta i$ is a **root** of the auxiliary equation, take $s = 1$.