Math 677: Homework Assignments

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Homeworks

Homework 1: Due September 2, 2009

§1.1: 9, 22, 25

§1.2: 3

§1.3: 15, 19

§1.4: 2, 11

§1.6: 2, 7, 17, 20

Bonus question: Given a permutation $\sigma \in S_n$ written as a product of disjoint cycles, state a simple criterion for deciding whether σ is a square in S_n (i.e., whether there is a permutation ρ such that $\sigma = \rho^2$.

Homework 2: Due September 9, 2009

§1.7: 16, 17

§2.1: 6, 8, 10

§2.2: 4, 6

§2.3: 1, 16, 24

§2.4: 8, 9, 11

Bonus question: (3 points)

(a) If a group G acts on a set S, and if T is a subset of S, show that the sets $\{g \in G \mid gT \subseteq T\}$ and $\{g \in G \mid gT \supseteq T\}$ are both closed under the group operation, and contain 1, and that the intersection, $\{g \in G \mid gT = T\}$, is a subgroup of G.

This leads us to wonder whether the two sets described above are themselves subgroups. We will answer this with the help of an example. For any two real numbers m and b with $m \neq 0$, let $a_{m,b}$ be the set-map $\mathbb{R} \to \mathbb{R}$ defined by $a_{m,b}(x) = mx + b$.

(b) Show that the set G of such maps forms a group under composition.

(c) Let us regard G as acting on $\mathbb{R}$ in the obvious way, by letting $a_{m,b} \cdot x = a_{m,b}(x)$. Determine the sets $\{g \in G \mid g\mathbb{Z} \subseteq \mathbb{Z}\}$ and $\{g \in G \mid g\mathbb{Z} \supseteq \mathbb{Z}\}$, and show that neither of them is a subgroup of G.

This example is also related to the normalizer construction:

(d) Let $t \in G$ denote the element $a_{1,1}$, and $H \leq G$ denote the subgroup $H = \langle t \rangle$. Show that the sets $\{g \in G \mid gHg^{-1} \subseteq H\}$ and $\{g \in G \mid gHg^{-1} \supseteq H\}$ also fail to be subgroups of G, but that their intersection is the subgroup $N_G(H)$. [Hint: Prove first that for any $a_{m,b} \in G$ and $r \in \mathbb{R}$, one has $a_{m,b}a_{1,r}(a_{m,b})^{-1} = a_{1,mr}$.]

Homework 3: Due September 16, 2009

§3.1: 1, 6, 16, 17, 35, 36

Bonus question: (2 points)

36 (b) Show by example that the condition "$G/Z(G)$ is Abelian" does not imply that G is Abelian. (You might find it interesting to examine how the proof for 36 fails to go over to this similar hypothesis.)

36 (c) Show that if $G/Z(G) \cong \mathbb{Q}$ the G is Abelian.

Homework 4: Due September 23, 2009

§3.2: 4, 5, 11, 16

§3.3: 3, 7

Homework 5: Due September 30, 2009

§3.4: 1, 4

§4.1: 1

§4.2: 3, 9

§4.3: 6, 23, 24 [* You can postpone problems 23, 24, and 27' in this section for next homework.]

27'. Let G be a finite group, and let $g_1, g_2, \ldots, g_r \in G$ be representatives of its nonidentity conjugacy classes.

(a) Show that $\langle g_1, g_2, \ldots, g_r \rangle = G$. [Hint: Apply Exercise 24 to $\langle g_1, g_2, \ldots, g_r \rangle$.]

(b) Deduce the result of Exercise 27 as stated in the text.

37. Let G be a group of order 21 with $Z(G) = 1$. Prove that G has exactly 5 conjugacy classes.

Homework 6: Due October 12, 2009

§4.4: 1, 3, 6

§4.5: (Choose exactly 2 of each set) $\{1, 4, 5\}, \{13, 16, 18\}, \{24, 33\}$

Homework 7: Due October 23, 2009

§5.1: 1, 4, 7, 11, 14

§5.2: 4c, 7, 8

Homework 8: Due October 30, 2009

§5.4: 2, 4, 5, 19a

§5.5: 1, 8, 12

Homework 9: Due November 6, 2009

§7.1: 6, 7, 13, 14

§7.2: 1, 3, 7, 10, (read 5)

Homework 10: Due November 18, 2009

§7.3: 4*, 17, 21, 24*, 29, 34 (* means problem is optional)

§7.4: 11, 13, 15, 19*, 26, 30*

§7.5: 3

Homework 11: Due December 2, 2009

§8.1: 3, 7, 8a, 10

§8.2: 3, 6

§8.3: 5, 11

Homework 12: Due December 9, 2009

§9.1: 7, 11, 12

§9.2: 1, 3, 7

§9.3: 1, 4, (read 5)