MTH 376
Homework 7

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Section 3.3

4. The IVP is

$$T' = K(23 - T), \qquad T(0) = 10,$$

the constant $K = -\frac{1}{10}\ln(\frac{8}{13})$, and solving the equation $T(t) = 18$ gives $t \approx 19.68$ min.

6. For the first part, the IVP is

$$T' = \frac{1}{3}[12 - T], \qquad T(0) = 21,$$

so $t = 2.4328$ or 2 hours with 26 minutes. For the second part, the IVP is

$$T' = \frac{1}{2}[12 - T], \qquad T(0) = 21,$$

so $t = 1.622$ or 1 hour with 37 minutes.

10. The differential equation is $T' = K[M(t) - T(t)] + K_U[T_D - T(t)]$ (see Example 3 in page 105). Here T_D is the desired temperature and the proportionality constant K_U satisfies the equation $K_1 = K + K_U$, where $1/K$ is the usual time constant and $1/K_1$ is the time constant for the building with heating and air conditioning (in this case $2 = \frac{1}{2} + K_U$) . So the IVP is

$$T' = \frac{1}{2}[40 - T] + \frac{3}{2}[70 - T], \qquad T(0) = 40.$$

The differential equation simplifies to $T' = 125 - 2T$. The temperature at 8 a.m. is $T(1) = 59.455°$ F. Moreover, the limit as $t \to \infty$ of $T(t)$ equals 62.5, so the temperature inside the hall will never reach 65° F.

Section 3.4

8. This problem has to be divided in two parts. First a free-fall part (before the chute is open). The form of the the DE is $m\frac{dv}{dt} = mg - bv$. Note that the gravitational force has the same direction as the displacement but air resistence goes in the opposite direction. In this part, $m = 100$, $b = 20$, and $v(0) = 0$. So the IVP is

$$v' = g - .2v, \qquad v(0) = 0.$$

Solving we obtain $v(t) = 49.05 - 49.05e^{-t/5}$. We integrate to obtain the equation for displacement

$$x(t) = 245.25e^{-t/5} + 49.05t - 245.25.$$

Note that the constant term was obtained by substituting $x(0) = 0$. The chute opens after 30 seconds, at this moment we have

$$x(30) \approx 1227, \qquad v(30) = 48.93$$

In the second part, we need to compute the equation for displacement once the chute opens. The constant b changes to $b = 100$, so the corresponding IVP is $v' = g - v$ with IC $v(0) = 48.93$ as computed above. We solve to obtain the equation for velocity $v(t)$ and then integrate to obtain $x(t)$, using that $x(0) = 0$ we get

$$x(t) = -39.12e^{-t} + 9.81t + 39.12.$$

Finally, we need to solve for t in the equation $x(t) = 3000 - 1227 = 1773$. Note that you can do this by hand by assuming that the exponential term has died off (it is so small that we can take it to be 0). So $9.81t + 39.12 = 1773$, and $t \approx 176.75$. Hence, the total amount of time to reach the ground is $176.75 + 30 = 206.75$ seconds.