Section 3.3

4. The IVP is

\[ T' = K(23 - T), \quad T(0) = 10, \]

the constant \( K = -\frac{1}{10} \ln \left( \frac{8}{13} \right) \), and solving the equation \( T(t) = 18 \) gives \( t \approx 19.68 \text{ min} \).

6. For the first part, the IVP is

\[ T' = \frac{1}{3}[12 - T], \quad T(0) = 21, \]

so \( t = 2.4328 \) or 2 hours with 26 minutes. For the second part, the IVP is

\[ T' = \frac{1}{2}[12 - T], \quad T(0) = 21, \]

so \( t = 1.622 \) or 1 hour with 37 minutes.

10. The differential equation is \( T' = K[M(t) - T(t)] + K_U[T_D - T(t)] \) (see Example 3 in page 105). Here \( T_D \) is the desired temperature and the proportionality constant \( K_U \) satisfies the equation \( 1/K_1 = K + K_U \), where \( 1/K \) is the usual time constant and \( 1/K_1 \) is the time constant for the building with heating and air conditioning (in this case \( 2 = \frac{1}{2} + K_U \)). So the IVP is

\[ T' = \frac{1}{2}[40 - T] + \frac{3}{2}[70 - T], \quad T(0) = 40. \]

The differential equation simplifies to \( T' = 125 - 2T \). The temperature at 8 a.m. is \( T(1) = 59.455^\circ \text{ F} \). Moreover, the limit as \( t \to \infty \) of \( T(t) \) equals 62.5, so the temperature inside the hall will never reach 65° F.
Section 3.4

8. This problem has to be divided in two parts. First a free-fall part (before the chute is open).

The form of the DE is \( m \frac{dv}{dt} = mg - bv \). Note that the gravitational force has the same
direction as the displacement but air resistance goes in the opposite direction. In this part,
\( m = 100, b = 20, \) and \( v(0) = 0 \). So the IVP is
\[
    v' = g - .2v, \quad v(0) = 0.
\]

Solving we obtain \( v(t) = 49.05 - 49.05e^{-t/5} \). We integrate to obtain the equation for displacement
\[
    x(t) = 245.25e^{-t/5} + 49.05t - 245.25.
\]

Note that the constant term was obtained by substituting \( x(0) = 0 \). The chute opens after 30
seconds, at this moment we have
\[
    x(30) \approx 1227, \quad v(30) = 48.93
\]

In the second part, we need to compute the equation for displacement once the chute opens. The
constant \( b \) changes to \( b = 100 \), so the corresponding IVP is \( v' = g - v \) with IC \( v(0) = 48.93 \) as
computed above. We solve to obtain the equation for velocity \( v(t) \) and then integrate to obtain
\( x(t) \), using that \( x(0) = 0 \) we get
\[
    x(t) = -39.12e^{-t} + 9.81t + 39.12.
\]

Finally, we need to solve for \( t \) in the equation \( x(t) = 3000 - 1227 = 1773 \). Note that you can do
this by hand by assuming that the exponential term has died off (it is so small that we can take
it to be 0). So \( 9.81t + 39.12 = 1773 \), and \( t \approx 176.75 \). Hence, the total amount of time to reach
the ground is \( 176.75 + 30 = 206.75 \) seconds.