§ 3.2

1. $\frac{dx}{dt} = (8)(.05) - \frac{8}{100} x(t)$

$x' + .08x = .4 \qquad x(0) = .5$

$x = 5 - 4.5 e^{-2t/25}$

$2 = 5 - 4.5 e^{-2t/25} \qquad t = 5.068 \text{ min}$

4. $\frac{dx}{dt} = (4)(.2) - 3\frac{x(t)}{100+t} \qquad x(0) = 0$

$x = -\frac{2 \times 10^7}{(100+t)^3} + 20 + \frac{1}{5} t$

$\frac{x(t)}{100+t} = .1 \qquad x(t) = .1(100+t)$

$t = 18.92$

14. $p(t) = p_0 e^{kt} \qquad p_0 = p(0) = 300 \qquad p(10) = 1500 \qquad 1500 = 300 e^{10k}$

$k = 0.161$

$p(40) = 300 e^{(.161)(40)} = 187,500$

15. $p_0 = p(0) = 300 \qquad p_a = p(5) = 1200 \qquad p_b = p(10) = 1500$

$p_1 = \left[\frac{p_a p_b - 2 p_a p_b + p_0 p_a}{p_a^2 - p_0 p_b} \right] p_a = 1527.27$

$A = \frac{1}{p_1 t_a} \ln \left[\frac{p_b (p_a - p_0)}{p_0 (p_b - p_a)} \right] = 3.5462 \times 10^{-4}$

$p(t) = \frac{p_0 p_1}{p_0 + (p_1 - p_0) e^{-A p_1 t}} = \frac{458181.81}{300 + 1227.2727 e^{-.5416 t}}$

$p(40) = 1527.27 \qquad \lim_{t \to \infty} p(t) = \frac{p_0 p_1}{p_0} = p_1$

~~17.~~ (a) $\frac{1}{P(t)}\frac{dP}{dt} \approx \frac{1}{P(t)}\frac{P(t+10)-P(t)}{10}$ .0351

.03107

.0302

25. half-life carbon-14 is 5600 years

$P(t) = P_0 e^{kt}$ $\frac{P_0}{2} = P_0 e^{k5600}$, $\frac{1}{2} = e^{5600k}$, $\frac{\ln(\frac{1}{2})}{5600} = k = -1.237$

$.02P_0 = P_0 e^{kt}$, $e^{kt} = .02$, $\frac{\ln(.02)}{k} = t$ $k = -1.2377e^{-4}$

$t = 31,605.59$ 31,606 years

26 (a) 5550 years $k = \frac{\ln(\frac{1}{2})}{5550}$ $t = \frac{\ln(.02)}{k} = 31,323$ years

(b) $t = \frac{\ln(.03)}{k} = 28076.85$ years $t = \frac{\ln(.03)5600}{\ln(.5)} = 28330$

(c) mass-remaining.

§ 3.3

4. $\frac{dT}{dt} = k[M(t) - T(t)]$ $\quad T' = k(23 - T)$, $T(0) = 10$

$T(t) = 23 - 13e^{-kt}$, $\quad 15 = 23 - 13e^{-k\cdot 10}$, $\quad k = -\frac{1}{10}\ln\left(\frac{8}{13}\right)$

$T(t) = 18$ $\quad t = \frac{10 \ln \frac{5}{13}}{\ln \frac{8}{13}} \approx 19.68$ min.

(5.) $T' = k(16 - T)$ $\quad T(0) = 37.5$ $\quad T' + kT = 16k$

$T(t) = 16 + 37/2\, e^{-kt}$

$T(1) = 33.7$ $\quad k = .442061$

$37 = 16 + 37/2\, e^{-.044206\, t}$ $\quad t = -2.8673$ $\quad$ 52 min

9:08 am

6. $T' = k[M(t) - T(t)] + H(t) + u(t)$

$T' = \frac{1}{3}[12 - T]$ $\quad T(0) = 21°C$

$T(t) = 12 + 9e^{-t/3}$

$16 = 12 + 9e^{-t/3}$ $\quad t = -3\log(4/9) = 2.4328$ $\quad$ 2 hr 26 min

$T' = \frac{1}{2}[12 - T]$ $\quad T(0) = 21$

$T(t) = 12 + 9e^{-t/2}$ $\quad t = -2\ln(4/9) = 1.622$ $\quad$ 1 hr 37 min

* 10. $M(t) = 40$ $\quad T(0) = 40$ $\quad k = 1/2$ $\quad \frac{1}{k_1} = \frac{1}{2}$ $\quad k_1 = 2$

$\frac{1}{2} + k_u = 2$

$T' = k[M(t) - T(t)] + k_u[T_D - T(t)]$

$T' = \frac{1}{2}[40 - T] + \frac{3}{2}[70 - T]$ $\quad T(0) = 40$

$T' = 125 - 2T$

$T = 125/2 - 45/2\, e^{-2T}$

$T(1) = 59.455\ °F$

$T = 65$ $\quad t =$ never

$\lim_{t \to \infty} T(t) = \frac{125}{2} = 62.5$

(11.) $T'(t) = \frac{1}{2}(35 - T) + k_u(T_0 - T)$ $\quad \frac{1}{k_1} = \frac{1}{3}$ $\quad k_1 = 3$ $\quad \frac{1}{2} + k_u = 3$

$T' = \frac{1}{2}(35 - T) + \frac{5}{2}(16 - T)$

$T' = 57.5 - 3T \qquad T(0) = 55$

$T = 115/6 + 215/6\, e^{-3t}$

$27 = 115/6 + 215/6\, e^{-3t} \qquad t = 30.41$ ~~sec~~ min

206.8 sec 86.8 sec

§ 3.4

3000 m

free-fall 30 sec

1 min

$b_3 = 20$

$b_4 = 100$

8. $m = 100$ kg

free-fall: $v(0) = 0$

$$m\frac{dv}{dt} = mg - bv$$

$$v' = g - .2v \quad , \quad v = 981/20 - 981/20\, e^{-t/5}$$

$$x = 981/4\, e^{-t/5} + 981/10\, t - 981/4$$

$x(30) = 1226.9$ 1227 m

$v(30) = 48.93$ m/sec

after chute open

$$mv' = mg - bv$$

$v' = g - v \quad v(0) = 48.93$

$x' = v = 981/100 + 24449/625\, e^{-t} \quad x(0) = 0$

$x = -24449/625\, e^{-t} + 981/100\, t + 24449/625$

$1773 = 3000 - 1227 = x(t)$

$1773 = 9.81t + 24449/625$

$t \approx 176.75$ sec 1768 sec

same thing 1 min +30

206.8 sec

9. $m = 100\ kg$

$$mv' = mg - \frac{1}{40}mg - 10v$$

$$v' = \frac{39}{40}g - .1v \qquad v(0) = 0$$

$$\rightarrow v = 38259/400 - 38259/400\, e^{-t/10} \qquad x(0) = 0$$

$$x = 38259/40\, e^{-t/10} + 38259/400\, t - 38259/40$$

$$v = 70 \qquad t = -10 \ln(10259/38259) = 13.162$$