

# MATH 142 — EXAM 3 FORM A

DEPARTMENT OF MATHEMATICS  
Texas A & M University

November 21, 2005

NAME: Key SECTION: \_\_\_\_\_

SIGNATURE: \_\_\_\_\_

**Part 1. (Problems 1-8). Multiple Choice.** Clearly mark your answers on the ScanTron form using a #2 pencil. Record your answers on your exam for your own records. Remember to print your name on your ScanTron form along with 1) Exam 1 Form (A or B) and 2) Math 142 Section 501 or 508

**Part 3. (Problems 9-13). Work Out.** Write solutions in the space provided. Clearly indicate final answers.

PROBLEM	POINTS	SCORE
1-8	40	
9	10	
10	10	
11	10	
12	10	
13	15	
14	5	
TOTAL	100	

**Part 1. Multiple Choice** Circle the correct answer, and then mark the corresponding letter on your ScanTron form. No partial credit. Each problem is worth 5 points.

1. Compute  $\int_5^{10} (4 - 2x + 8x^2) dx$

(a) 2005

(b) 2428

(c) 2270

(d) 2278 ← counted as a correct answer. Even though it is just an approximation.

(e) none of these

2. The area between  $y = x^2 - 4x - 4$  and  $y = x - 4$  on  $[0, 5]$  is given by the formula

(a)  $\int_0^1 (x - 4) - (x^2 - 4x - 4) dx$

(b)  $\int_0^5 (x^2 - 4x - 4) - (x - 4) dx$

(c)  $\int_0^5 (x - 4) - (x^2 - 4x - 4) dx$

(d)  $\int_0^5 (x^2 - 4x - 4) + (x - 4) dx$

(e) none of these

3. You are given that marginal profit is  $MP(x) = 50 - .02x$ . The units of  $MP$  are dollars per item. If you know that  $P(10) = 450$  then the profit function,  $P$ , is

(a)  $P(x) = 50x - .02x^2$

$$50x - .01x^2 + C$$

(b)  $P(x) = 50x - .01x^2$

$$500 - 1 + C = 450$$

(c)  $P(x) = 50x - .01x^2 - 40$

$$499 + C = 450$$

(d)  $P(x) = 50x - .02x^2 - 49$

$$C = -49$$

(e) none of these

$$50x - .01x^2 - 49$$

4. Let  $f(x) = e^{2x} + 1$ . The equation for the tangent line to the graph of  $f$  at  $x = 1$  is

(a)  $y = 2e^2x - e^2 - 1$

$$f' = 2e^{2x} \quad f'(1) = 2e^2$$

(b)  $y = e^2x - e^2 + 1$

$$f(1) = e^2 + 1$$

(c)  $y = 2e^2x - e^2 + 1$

$$y - e^2 - 1 = 2e^2(x - 1) = 2e^2x - 2e^2$$

(d)  $y = e^2x - e^2 - 1$

$$y = 2e^2x - e^2 + 1$$

(e) none of these

$$5. \lim_{x \rightarrow -\infty} \frac{3^x - 2x}{5^x + 2} = \frac{\infty}{2}$$

(a)  $\infty$

(b)  $-\infty$

(c)  $\frac{3}{5}$

(d) 0

(e) none of these

$$\lim_{x \rightarrow -\infty} 3^x = \lim_{x \rightarrow -\infty} 5^x = 0$$

$$\lim_{x \rightarrow -\infty} -2x = \infty$$

Questions 6 and 7: An oil field has a production rate (in thousands of barrels per year) of  $R(t) = \frac{100t}{t^2+1} + 5$  where  $t$  is the number of years since pumping began.

6. How many barrels of oil have been produced during the first twenty years? (Round to the nearest barrel)

(a) 4,988 barrels

(b) 19,985 barrels

(c) 9,988 barrels

(d) 399,698 barrels

(e) none of these

$$\int_0^{20} \left( \frac{100t}{t^2+1} + 5 \right) dt * 1000$$

7. What is the average production rate during the first twenty years? (Round to the nearest barrel)

(a) 399,698 barrels

(b) 499 barrels

(c) 19,985 barrels

(d) 249 barrels

(e) none of these

$$\frac{399,698}{20 - 0} \approx 19,985$$

8. Find the instantaneous rate of change at  $x = 2$  if  $f(x) = 5^{4x^2-6x}$ .

(a) 625

(b)  $6,250(\ln 5)$

(c) 6250

(d)  $10(\ln 5)$

(e) none of these

$$5^{4x^2-6x} \cdot (8x-6) \cdot \ln(5)$$

$$5^{16-12} \cdot 10 \cdot \ln(5)$$

$$5^4 \cdot 10 \cdot \ln(5)$$

Part 2. Work Out Circle final answers.

In Problems 9 and 10, find the integrals.

9. (10 points)

$$\int 3x + \frac{1}{x} - e^{5x} + 2^x dx$$

$$\frac{3}{2}x^2 + \ln|x| - \frac{e^{5x}}{5} + \frac{2^x}{\ln(2)} + C$$

10. (10 points)

$$\int x(x^2+1)^3 dx$$

$$u = x^2 + 1$$

$$du = 2x dx$$

$$\frac{du}{2} = x dx$$

$$\frac{1}{2} \int u^3 du = \frac{1}{2} \frac{u^4}{4} + C$$

$$= \frac{1}{8} u^4 + C$$

$$= \frac{1}{8} (x^2+1)^4 + C$$

11. Compute the derivatives of the following functions. Do not simplify.

(a) (5 points)  $f(x) = e^{(6x^2 - 8x + 4)^4}$ .

$$f'(x) = e^{(6x^2 - 8x + 4)^4} \cdot 4(6x^2 - 8x + 4)^3 \cdot (12x - 8)$$

(b) (5 points)  $f(x) = \ln((3x^2 - 8x)(\log_3(x) + 8x + 5))$ .

$$f(x) = \ln(3x^2 - 8x) + \ln(\log_3 x + 8x + 5)$$

$$f'(x) = \frac{1}{3x^2 - 8x} (6x - 8) + \frac{1}{\log_3 x + 8x + 5} \left( \frac{1}{x} \cdot \frac{1}{\ln(3)} + 8 \right)$$

OR

$$f'(x) = \frac{1}{(3x^2 - 8x)(\log_3 x + 8x + 5)} \cdot \left[ (6x - 8)(\log_3 x + 8x + 5) + (3x^2 - 8x) \left( \frac{1}{\ln(3)} \cdot \frac{1}{x} + 8 \right) \right]$$

12. (10 points) A contaminant has a half life of 80 years. If 100 grams are present, how long will it take for the contaminant to break down to 40 grams?

$$F = P e^{rt}$$

$$50 = 100 e^{r \cdot 80}$$

$$.5 = e^{r \cdot 80}$$

$$80r = \ln(.5)$$

$$r = \frac{\ln(.5)}{80} = -.0087$$

$$40 = 100 e^{-.0087t}$$

$$.4 = e^{-.0087t}$$

$$-.0087t = \ln(.4)$$

$$t = \frac{\ln(.4)}{-.0087} \approx 105$$

between 105 and 106 years

13. If the demand equation is  $d(x) = 25 - .004x^2$  and the supply equation is  $s(x) = 5 + .004x^2$ , find

(a) (5 points) the equilibrium point.

$$25 - .004x^2 = 5 + .004x^2$$

$$20 = .008x^2$$

$$x^2 = 2500 \quad (50, 15)$$

$$x = \pm 50$$

(b) (5 points) the consumers' surplus.

$$\int_0^{50} (25 - .004x^2 - 15) dx = 333.33$$

(c) (5 points) the producer's surplus.

$$\int_0^{50} (15 - (5 + .004x^2)) dx = 333.33$$

14. (5 points) State the Fundamental Theorem of Calculus. **Do Not** forget to include the hypothesis on the function and the interval. In other words, if you just write a formula, you will not get full credit.

Let  $f(x)$  be a continuous function on  $[a, b]$  and

$F(x)$  be an antiderivative of  $f(x)$ . Then

$$\int_a^b f(x) dx = F(b) - F(a).$$

# MATH 142 — EXAM 3 FORM B

DEPARTMENT OF MATHEMATICS  
Texas A & M University

November 21, 2005

NAME: Key SECTION: \_\_\_\_\_

SIGNATURE: \_\_\_\_\_

**Part 1. (Problems 1-8). Multiple Choice.** Clearly mark your answers on the ScanTron form using a #2 pencil. Record your answers on your exam for your own records. Remember to print your name on your ScanTron form along with 1) Exam 1 Form (A or B) and 2) Math 142 Section 501 or 508

**Part 3. (Problems 9-13). Work Out.** Write solutions in the space provided. Clearly indicate final answers.

PROBLEM	POINTS	SCORE
1-8	40	
9	10	
10	10	
11	10	
12	10	
13	15	
14	5	
TOTAL	100	

**Part 1. Multiple Choice** Circle the correct answer, and then mark the corresponding letter on your ScanTron form. No partial credit. Each problem is worth 5 points.

1. Find the instantaneous rate of change at  $x = 2$  if  $f(x) = 5^{4x^2-6x}$ .

- (a) 625
- (b)  $6,250(\ln 5)$
- (c) 6250
- (d)  $10(\ln 5)$
- (e) none of these

2.  $\lim_{x \rightarrow -\infty} \frac{3^x - 2x}{5^x + 2} =$

- (a)  $\infty$
- (b)  $-\infty$
- (c)  $\frac{3}{5}$
- (d) 0
- (e) none of these

Questions 3 and 4: An oil field has a production rate (in thousands of barrels per year) of  $R(t) = \frac{100t}{t^2+1} + 5$  where  $t$  is the number of years since pumping began.

3. How many barrels of oil have been produced during the first twenty years? (Round to the nearest barrel)

- (a) 4,988 barrels
- (b) 19,985 barrels
- (c) 9,988 barrels
- (d) 399,698 barrels
- (e) none of these

4. What is the average production rate during the first twenty years? (Round to the nearest barrel)

- (a) 399,698 barrels
- (b) 499 barrels
- (c) 19,985 barrels
- (d) 249 barrels
- (e) none of these

5. Let  $f(x) = e^{2x} + 1$ . The equation for the tangent line to the graph of  $f$  at  $x = 1$  is

(a)  $y = 2e^2x - e^2 - 1$

(b)  $y = e^2x - e^2 + 1$

(c)  $y = 2e^2x - e^2 + 1$

(d)  $y = e^2x - e^2 - 1$

(e) none of these

6. The area between  $y = x^2 - 4x - 4$  and  $y = x - 4$  on  $[0, 5]$  is given by the formula

(a)  $\int_0^1 (x - 4) - (x^2 - 4x - 4) dx$

(b)  $\int_0^5 (x^2 - 4x - 4) - (x - 4) dx$

(c)  $\int_0^5 (x - 4) - (x^2 - 4x - 4) dx$

(d)  $\int_0^5 (x^2 - 4x - 4) + (x - 4) dx$

(e) none of these

7. You are given that marginal profit is  $MP(x) = 50 - .02x$ . The units of  $MP$  are dollars per item. If you know that  $P(10) = 450$  then the profit function,  $P$ , is

(a)  $P(x) = 50x - .02x^2$

(b)  $P(x) = 50x - .01x^2$

(c)  $P(x) = 50x - .01x^2 - 40$

(d)  $P(x) = 50x - .02x^2 - 49$

(e) none of these

8. Compute  $\int_5^{10} (4 - 2x + 8x^2) dx$

(a) 2005

(b) 2428

(c) 2270

(d) 2278

(e) none of these

**Part 2. Work Out** *Circle final answers.*

In Problems 9 and 10, find the integrals.

9. (10 points)

$$\int 3x + \frac{1}{x} - e^{5x} + 2^x dx$$

10. (10 points)

$$\int x(x^2 + 1)^3 dx$$