MATH 142 — EXAM 2 FORM A

DEPARTMENT OF MATHEMATICS
Texas A & M University

October 26, 2005

NAME: ______________________ SECTION: ______________________

SIGNATURE: ________________

Part 1. (Problems 1-10). Multiple choice. Clearly mark your answers on the ScanTron form using a #2 pencil. Record your answers on your exam for your own records. Remember to print your name on your ScanTron form along with 1) Exam 2 Form (A or B) and 2) Math 142 Section 501 or 508.

Part 2. (Problems 11-15). Work Out. Write solutions in the space provided. Clearly indicate final answers. Show the work done to obtain your answer.

PROBLEM	POINTS	SCORE
1-10	50	
11	10	
12	10	
13	20	
14	10	
TOTAL	100	

Part 1. Multiple Choice *Circle the correct answer, and then mark the corresponding letter on your ScanTron form. No partial credit. Each problem is worth 5 points.*

1. Find $\lim_{x \to \infty} \frac{4x^3+4x+3}{3x^3+2x+3}$

(a) ∞

(b) 0

(c) 4/3

(d) 1

(e) none of these

2. Let $f(x) = x^2 - 5x + 3$. The equation for the tangent line to the graph of f at $x = 1$ is

(a) $y = 2x - 1$

(b) $y = -3x + 4$

(c) $y = 3x - 4$

(d) $y = -3x + 2$

(e) none of these

$f'(x) = 2x - 5$

$f'(1) = -3$, $f(1) = -1$

$y + 1 = -3x + 3$

$y = -3 + 2$

3. What is the absolute minimum value of $f(x) = 2x^3 - 3x^2 - 12x + 24$ on $[-2, 3]$.

(a) 2

(b) -2

(c) 4

(d) there is no absolute minimum

(e) none of these

$f'(x) = 6x^2 - 6x - 12 = 6(x-2)(x+1)$

$f'(x) = 0$ if $x = 2, -1$

x	$f(x)$
-2	20
-1	31
2	4
3	15

min @ $x = 2$

min value $f(2)$

4. Find $\lim_{x \to 6} \frac{x^2-36}{x+6}$

(a) 0

(b) ∞

(c) undefined

(d) -12

(e) none of these

$\frac{x^2-36}{x+6}(6) = \frac{0}{12} = 0$

5. Define $f(x)$ by

$$f(x) = \begin{cases} x^{1/3} & x \le 1 \\ 1 & 1 < x \le 2 \\ x^3 & x > 2 \end{cases}$$

This function is not continuous at

(a) $x = 1,\ 2$

(b) $x = 2$

(c) $x = 1$

(d) $x = 0,\ 1$

(e) none of these

$x^{1/3}(1) = 1$ so $f(x)$ cont. at 1.

$x^3(2) = 2^3 = 8 \neq 1$ so $f(x)$ <u>not</u> cont at 2.

6. The function in Question 5 is not differentiable at

(a) $x = 0,\ 2$

(b) $x = 1,\ 2$

(c) $x = 0,\ 1$

(d) $x = 0,\ 1,\ 2$

(e) none of these

@ 2 not continuous

@ 1 sharp edge

@ 0 vertical tangent $(x^{1/3})' = \frac{1}{3}\frac{1}{x^{2/3}}$ not def at 0.

7. Find the average rate of change from $x = 30$ to $x = 40$ when $f(x) = -5x^2 + 800x$.

(a) 450

(b) 400

(c) 500

(d) 100

(e) none of these

$$\frac{f(40) - f(30)}{40 - 30} = \frac{24000 - 19500}{10} = 450$$

8. Find $\lim_{x \to -2} \frac{x^2-4}{x+2} = \lim_{x \to -2} \frac{(x-2)(x+2)}{(x+2)} = \lim_{x \to -2} x - 2 = -4$

(a) undefined

(b) -4

(c) ∞

(d) 0

(e) none of these

9. What is the absolute maximum value of $f(x) = x + \frac{9}{x}$ on $(-\infty, 0)$.

(a) 3

(b) -6

(c) -3

(d) there is no absolute maximum

(e) none of these

$f'(x) = 1 - \frac{9}{x^2}$ or $f'(x) = \frac{x^2-9}{x^2}$ $f'(x)=0$ if $x=-3, 3$

only critical point in $(-\infty,0)$ $x=-3$

$f''(x) = \frac{18}{x^3}$ so $f''(-3) = -\frac{18}{27}$ $\Rightarrow$ max at $x=-3$

max value $f(-3)=-6$.

10. The function modelling car sales in College Station is $S(t)$, where t is years after 2000, and the units are thousands of cars. Which of the following is the correct interpretation of $S'(3) = 1.7$?

(a) Roughly 1,700 cars were sold in College Station in 2003.

(b) Sales of cars in College Station increased by 1,700 from 2002 to 2003.

(d) The number of cars sold in College Station increases by 1,700 cars each year after 2003.

(d) In 2003 the number of cars sold in College Station was increasing by roughly 1,700 cars per year.

(e) none of these

Part 2. Work Out Circle final answers. Show the work done to obtain your answer

11. (10 points) Iggy is still doing business analysis for his company. His cost function is $C(x) = 2000 + 0.3\sqrt{x}$, where x is the number of units produced, and $C(x)$ is dollars. Round to 3^{rd} dec.

(a) (2 point) What is the cost to produce 200 items?

$$C(200) = 2000 + .3\sqrt{200} = 2004.243$$

(b) (4 points) What is his marginal cost function? Compute $MC(25)$.

$$MC = C'(x) = \frac{.3}{2\sqrt{x}} = \boxed{\frac{0.15}{\sqrt{x}}}, \quad MC(25) = \frac{.15}{5} = .03$$

$$(MC = .15x^{-1/2})$$

(c) (4 points) What is his average cost function? Compute $AC(25)$.

$$AC = \frac{C(x)}{x} = \boxed{\frac{2000}{x} + \frac{.3\sqrt{x}}{x}} = \boxed{\frac{2000}{x} + \frac{.3}{\sqrt{x}}} = \boxed{\frac{2000}{x} + .3x^{-1/2}}$$

$$AC(25) = \frac{2000}{25} + \frac{.3(5)}{25} = 80 + .06 = 80.06$$

12. Compute the following derivatives. **Do not simplify.**

(a) (5 points)

$$f(x) = \left(9x^{-3} + 8x^{-2}\right)^5$$

$$f'(x) = 5\left(9x^{-3} + 8x^{-2}\right)^4 \cdot \left(-27x^{-4} - 16x^{-3}\right)$$

(b) (5 points)

$$g(x) = \frac{8x^3 - 9x^2 + 5^4}{x^{5/2} + 3\sqrt[4]{x} + 7}$$

$$\frac{\left(x^{5/2} + 3\sqrt[4]{x} + 7\right)\left(24x^2 - 18x\right) - \left(8x^3 - 9x^2 + 5^4\right)\left(\frac{5}{2}x^{3/2} + \frac{3}{4}x^{-3/4}\right)}{\left(x^{5/2} + 3\sqrt[4]{x} + 7\right)^2}$$

13.

(a) (10 points) If $f(x) = -x^3 - 3x^2 + 45x - 5$, find the critical values of f, the intervals where the function is increasing/decreasing and the relative extrema.

$f'(x) = -3x^2 - 6x + 45$

$f'(x) = 0 \quad -3x^2 - 6x + 45 = 0$

$-3(x+5)(x-3) = 0$

Crit. values $\boxed{x = -5,\ x = 3}$

f: dec, inc, dec

f': −, +, −

−5, 3

$f'(-6) = -27$

$f'(0) = 45$

$f'(4) = -27$

local extrema $\begin{cases} x = -5 & \text{local min} \\ x = 3 & \text{local max} \end{cases}$

(b) (10 points) If $f(x) = x^6 - 10x^4 - 5$, find the intervals where the function is concave up/down and the inflection points.

$f'(x) = 6x^5 - 40x^3$

$f''(x) = 30x^4 - 120x^2$

$= 30x^2(x^2 - 4)$

$f''(x) = 0$ when $x^2 = 0$ or $x^2 - 4 = 0$

$x = 0 \qquad x = \pm 2$

f: c. up, c. down, c. down, c. down

f'': +, −, −, +

−2, 0, 2

$f''(-3) = 1350 \qquad f(1) = -90$

$f''(-1) = -90 \qquad f(3) = 1350$

−2, 2 are inflection points

0 is not

14. (10 points) Use the following information to sketch $f(x)$.

- $\lim_{x\to\infty} f(x) = 8$
- $\lim_{x\to-\infty} f(x) = -5$
- $f(1) = 3$ and $f(4) = 5$ and $f(0) = 0$ and $f(5) = 6$
- vertical asymptote at $x = 2$
- $f'(x) > 0$ on $(-\infty, 1) \bigcup (4, \infty)$
- $f'(x) < 0$ on $(1, 2) \bigcup (2, 4)$
- $f''(x) > 0$ on $(-\infty, 0) \bigcup (2, 5)$
- $f''(x) < 0$ on $(0, 2) \bigcup (5, \infty)$