Randomly Decomposable Graphs in $2K_nP_4$

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This is the second project I researched in the NSF REU program. The authors of this paper are Dr. Ken Smith and I.

1 Introduction

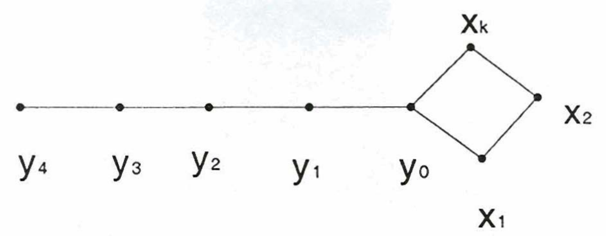
A graph G consists of a finite vertex set $V(G)$ and a symmetric antireflexive relation $E(G)$. These graphs are labeled as "simple" and "undirected". If G and H are graphs, we refer to a subgraph of G that is isomorphic to H as a H-subgraph of G. If H is a subgraph, we define $G - H$ to be the graph obtained by first forming the graph with vertex set $V(G)$ and edge set $E(G) - E(H)$, and then deleting from this graph any isolated vertices.

Definition 1 An H-decomposition of G is a family of edge disjoint H-subgraphs of G whose union is G. When this occurs, we say G is H-decomposable and write "H/G". Given a graph H, set $D(H) := G : H|G$.

Definition 2 We say G is randomly H-decomposable if any edge disjoint family of H-subgraphs of G can be extended to an H-decomposition of G. The set of all randomly H-decomposable graphs is denoted RD(H).

2 Graph Labeling

The following graph describes the labeling used to examine $2K_nP_k$ graphs. The concentration of this paper will be on the specific case of $2K_nP_4$ graphs. The labeling of the path connected to the complete part will (in this case) be y_4, y_3, y_2, y_1, y_0. The complete part of the graph will depend on the number of edges or what k is. A single graph H will be combined with an identical graph H'. When the two copies of H are combined, they connect at a certain vertex and share that vertex. There exist many combination of vertices from one vertex shared to many G graphs from sharing one vertex to sharing many vertices.



3 Randomly Decomposable Graphs With One Vertex Shared

Randomly decomposable graphs exist when the vertices $y_j = y_k$ in the following cases:

1. $j = 0$ and $k = 0$
2. $j = 4$ and $k = 4$

4 Theorem 1.1

If there is a $2K_nP_4$ graph connected by on vertex, there are four cases where no randomly decomposable graphs exist. The $2K_nP_4$ graph will be identified by H and H'.

Case 1

When $x_j = x'_k$ with $x_j \in H$ and $x'_k \in H'$ and WLOG assume that $x'_k = x'_1$. Since $x_j \in K_n \in H$ this vertex will be the first vertex of the connecting path. The following path selected will be: $x'_1, x'_2, x'_3 \ldots x_k', y'_0, y'_1, y'_2, y'_3, y'_4$ which will contain the majority of K'_n.

Case 2

When $x_j = y'_k$ with $x_j \in H$ and $y'_k \in H'$ and WLOG assume that $x_j = x_1$. The vertex $x_1 \in K_n \in H$ therefore following path will be selected: $x_1, y'_4, y'_3, y'_2, y'_1, y'_0\ x'_k, x'_{k-1}, x'_{k-2 \ldots}, x'_{k-p}$ where $1 \geq p \geq k$

Case 3

When $y_0 = y_k$' and $k \neq 0$ with $y_0 \in H$ and $y'_k \in H'$. The vertex $y_0 \in K_n$ will be the first vertex in the path. The following path will be selected: $y_0, y'_4, y'_3, y'_2, y'_1, y'_0 \ldots x'_k, x'_{k-1}, x'_{k-2 \ldots}, x'_{k-p}$ where $1 \geq p \geq k$ which will lead into the K'_n portion of the graph. In the case where all of the y'_{0-4} occurs, this does not include the K'_n part of H' but will isolate the K'_n from the P_4 path it has to have.

Case 4

When $y_j = y_k$', y_j will initiate the path if j=0 and if $j \neq 0$ then the vertex y_j will be a vertex of the path from $K_n \in H$. The path will be in the following order depending on the y_j point picked: $y_0, y_1, y_2, y_3, y_4, y'_4, y'_3, y'_2, y'_1, y'_0, x'_k, x'_{k-1} \ldots x'_{k-p}$ where p $1 \geq p \geq k$ Some cases will lead to the path becoming pieces of K'_n and other combinations will lead to the remaining graph consisting of various pieces of the remaining K_nP_4.

5 Randomly Decomposable Cases

Case 1

In a $2K_nP_k$ graph $\exists$ a RDG of

$$y_p = y_0,\ y_k = y'_k$$

where $p \geq 1$ and $k \geq n + p$ in a K_nP_k graph

Case 2

In a $2K_nP_k$ graph $\exists$ a RDG of

$$y_0 = y'_k,\ y_{k-(2b)} = y'_{k-(2b)},\ y_k = y'_0$$

where k=2j, $n \geq 3$, $j \geq 1$,B=j-1 and when q is the number of vertices shared, q=j.

Case 3

In a $2K_nP_k$ graph $\exists$ a RDG of

$$y_0 = y_k{}', y_k = y_0{}'$$

where $n \geq 3$ and $k \geq 2$

Case 4

In a $2K_nP_k$ graph $\exists$ a RDG of

$$y_0 = y_k{}', x_i = y_k$$

where $k \geq 1, n \geq 3$ and i can be 1 to k

Case 5

In a $2K_nP_k$ graph $\exists$ a RDG of

$$y_k = x_i{}',\ x_i = y_k{}'$$

where $k \geq 1, n \geq 3$ and i can be 1 to k

Case 6

In a $2K_nP_k$ graph $\exists$ a RDG of

$$y_0 = y_0' \text{ and } y_0 = y_0', y_k = y_k'$$

where $k \geq n$ and $n \geq 3$

Case 7

In a $2K_nP_k$ graph $\exists$ a RDG of

$$y_k = y_k{}'$$

where $n \geq 3$ and $k \geq 1$

Case 8

In a $2K_nP_k$ graph $\exists$ a RDG of

$$y_0 = y'_0, y_{k-(2b)} = y'_{k-(2b)}, y_k = y'_k$$

where $k = 2j$ where $j \geq 1$, $k \geq n$,$n \geq 3$,b=j-1 and j vertices will be shared.

Case 9

In a $2K_nP_k$ graph $\exists$ a RDG of

$$y_0 = y'_k, y_{k-(2b)} = y'_{k-(2b)}, y_k = x'_i$$

where $k = 2j$, $j \geq 1$,$n \geq 3$,$k \geq (n+1)$ $b = j-1$,i can be 1 to k and j+1 is the number of vertices shared.

6 Final Note

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References

[1] Smith, K.W., "Randomly Decomposable Graphs", Preprint (2002).