

Sample Questions
Mini-Exam (Review of prerequisite knowledge for Calculus)
MTH 142, Spring 2008

HUMANS ONLY! Calculators are NOT allowed.

1. Change the following degrees to radians
 - (a) 180 degrees
 - (b) 60 degrees
 - (c) 45 degrees
 - (d) 10 degrees
 - (e) -240 degrees
 - (f) 150 degrees

2. Change the following radians to degrees
 - (a) π
 - (b) $\pi/3$
 - (c) $\pi/4$
 - (d) $\pi/18$
 - (e) $-4\pi/3$
 - (f) $5\pi/6$

3. Find the length of the arc of the circle
 - (a) if the radius is 2 feet and the arc marks out an angle of π radians
 - (b) if the radius is 20 feet and the arc marks out an angle of 3 radians
 - (c) if the radius is 3 meters and the arc marks out an angle of $4/3$ radians
 - (d) if the radius is 200 miles and the arc subtends an angle of 1.5 radians
 - (e) if the diameter is 60 feet and the arc subtends an angle of 1.5 radians
 - (f) if the diameter is 20 angstroms and the arc subtends an angle of 4 radians

4. Find the exact value of
 - (a) $\tan(\frac{\pi}{4})$
 - (b) $\sin(\frac{\pi}{6})$
 - (c) $\cos(\frac{\pi}{3})$
 - (d) $\sec(\frac{\pi}{3})$

(e) $\sin\left(\frac{\pi}{4}\right)$

(f) $\tan\left(\frac{\pi}{3}\right)$

5. Find the exact value of

(a) $\cos(\pi)$

(b) $\sin\left(-\frac{\pi}{2}\right)$

(c) $\sin\left(\frac{7\pi}{6}\right)$

(d) $\cos\left(-\frac{5\pi}{6}\right)$

(e) $\sin\left(\frac{3\pi}{4}\right)$

(f) $\cos(13\pi)$

6. Find the exact value of

(a) $\tan^{-1}(1)$

(b) $\sin^{-1}\left(\frac{1}{2}\right)$

(c) $\cos^{-1}\left(\frac{1}{2}\right)$

(d) $\sec^{-1}(2)$

(e) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(f) $\tan^{-1}(\sqrt{3})$

7. Find the exact value of

(a) $\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)$

(b) $\tan\left(\sin^{-1}\left(\frac{4}{3}\right)\right)$

(c) $\sin\left(\cos^{-1}\left(\frac{4}{3}\right)\right)$

(d) $\tan\left(\tan^{-1}\left(\frac{4}{3}\right)\right)$

(e) $\sin^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right)$

(f) $\cos^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right)$

8. Find the sine of the angle θ if

(a) $\cos(\theta) = \frac{2}{3}$

(b) $\cos(\theta) = \frac{1}{2}$

(c) $\cos(\theta) = \frac{2}{5}$

(d) $\cos(\theta) = .4$

(e) $\cos(\theta) = \frac{5}{13}$

(f) $\tan(\theta) = 1$

9. Simplify the following expressions

(a) $\sqrt{b^6}$

(b) $\frac{\sqrt[3]{b^6}}{b}$

(c) $\frac{\sqrt[3]{b^6} b^4}{b^2}$

(d) $\frac{\sqrt[3]{b^5} b^3}{b^{-2}}$

(e) $\frac{(x^6)^{\frac{1}{3}} x^{-2}}{x^4}$

(f) $\frac{\sqrt[3]{y^3}}{\sqrt{y^2}}$

10. Simplify:

(a) $\frac{2b^3+b^2}{b}$

(b) $\frac{b^3+b^2}{b+1}$

(c) $\frac{b^4+2b^3+b^2}{b+1}$

(d) $\frac{a^2-9}{a-3}$

(e) $\frac{x^4-4}{x^2+2}$

(f) $\frac{y-4}{\sqrt{y+2}}$

11. Since $2^{10} = 1024$ is approximately 1000, let's simplify computations by approximating 2^{10} as $1000 = 10^3$. (Computer scientists do this; they often speak of *kilo-* as 1000 when it is often really 2^{10} .)

(a) If $2^{10} \approx 10^3$ then approximate 2^{20} as a power of ten.

(b) If $2^{10} \approx 10^3$ then approximate 2^{30} as a power of ten.

(c) If $2^{10} \approx 10^3$ then approximate 2^{100} as a power of ten.

(d) If $2^{10} \approx 10^3$ then approximate $2^{2^{10}}$ as a power of ten.

(e) If $2^{10} \approx 10^3$ compute the logarithm, base 10, of 2^{40} .

(f) If $2^{10} \approx 10^3$ compute $\log_{10}(2^{10})$.

12. Find the exact value

(a) $\log_{10}(10^3)$

(b) $\log_2(\sqrt{2})$

(c) $\log_e(e^{10})$

(d) $\log_2(4^3)$

(e) $\log_2(4^{30})$

(f) $\log_{10}\left(\frac{1}{100}\right)$

13. Find the exact value

(a) $\log_b(b^3)$

(b) $\log_b(b^3b^5)$

(c) $\log_b\left(\frac{b^3}{b^5}\right)$

(d) $\log_b((b^3)^5)$

(e) $\log_e(e^{\sqrt{2}})$

(f) $10^{\log_{10}(5)}$

14. Given the functions f and g , below, find the composition function $f \circ g$. ($f \circ g(x)$ is the same as $f(g(x))$).

(a) $f(x) = x^2$; $g(x) = \sqrt{x}$.

(b) $f(x) = \sqrt{x}$; $g(x) = x^2$.

(c) $f(x) = x^2 - 1$; $g(x) = x + 2$.

(d) $f(x) = x + 2$; $g(x) = x^2 - 1$.

(e) $f(x) = e^x$; $g(x) = x^2$.

(f) $f(x) = x^2$; $g(x) = e^x$.

15. Find the inverse function of $f(x)$.

(a) $f(x) = x^3$.

(b) $f(x) = \sqrt[3]{x}$.

(c) $f(x) = 4x - 5$.

(d) $f(x) = x^2 - 4$.

(e) $f(x) = x^3 + 5$.

(f) $f(x) = \sqrt{x^3 + 5}$.

16. Find the inverse function of $f(x)$.

(a) $f(x) = 10^x$.

(b) $f(x) = \log_e(x)$

(c) $f(x) = e^{x^2}$.

(d) $f(x) = e^{x^2-5}$.

(e) $f(x) = \tan(x)$.

(f) $f(x) = \sin^{-1}(x)$.

17. Find the equation for the line

- (a) with slope 4 passing through the point (2,3)
- (b) with slope -5 passing through the point (1,7)
- (c) with slope 3 passing through the point (20,-8)
- (d) with slope -5 and y -intercept (0, 17).
- (e) with slope -5 and x -intercept (4, 0).
- (f) passing through the points (2, 3) and (3, 7).

18. Find the x -intercept of the line

- (a) $y = 4x - 13$
- (b) the line has slope 4 and passes through the point (2,3)
- (c) the line has slope -5 and passes through the point (1,7)
- (d) the line has slope 3 and passes through the point (20,-8)
- (e) the line has slope -5 and y -intercept (0, 17).
- (f) the line passes through the points (1, 7) and (2, 2).

19. Assume each function below is a linear function.

- (a) The slope of graph of $y = f(x)$ is 4 and $f(2) = 3$. Find $f(3)$.
- (b) The slope of graph of $y = f(x)$ is -5 and $f(1) = 7$. Find $f(3)$.
- (c) The slope of graph of $y = f(x)$ is 3 and $f(20) = -8$. Find $f(19)$.
- (d) The slope of graph of $y = f(x)$ is -5 and the y -intercept is (0, 17). Find $f(2)$.
- (e) The line $y = f(x)$ passes through the points (1, 7) and (2, 2). Find $f(3)$.
- (f) The line $y = f(x)$ passes through the points (100, 70) and (102, 78). Find $f(103)$.