PLEASE SIGN AND DATE:

Although I may have discussed this exam with others, the material I have submitted here is my own. I understand what I have written and I can verbally defend what I have written.

Exam 2. Part I

Calculators with a Computer Algebra System may NOT be used.
You will be graded on your work and communication of it.

1. (12 points; 3 pts. for each part.) The volume V of a sphere is related to the radius r by the formula $V = \frac{4}{3}\pi r^3$.

   (a) Find a formula for the differential dV in terms of r and dr.

   (b) Suppose the radius of a spherical balloon is about 4 meters. Use differentials to estimate the error in the volume of the balloon if the error in the radius is about 0.2 meters.

   (c) A spherical balloon is being filling with air. Write out a formula for the instantaneous rate of change of the volume (dV/dt) of the balloon.

   (d) Suppose the volume of the spherical balloon is increasing at the rate of 10 cubic meters per minute. Find the instantaneous rate of change of the radius of the balloon when its radius is 4 meters.

2. (12 pts; 2 pts. for (a) and 10 pts. for (b).)

Two boats meet in the middle of the ocean. At noon, one boat heads north at 30 miles per hour. The other boat stays at the meeting point for an extra two hours, then, at 2 PM, heads west at 20 miles per hour.

(a) At 4 PM, what is the distance between the boats?

(b) At 4 PM, how rapidly is the distance between the boats changing? (Show your work/explain your reasoning.)

3. (SKIP) The graph of $y = f(x)$ is given below. Using that graph, draw in the space provided below it, the graphs of the first and second derivative.

4. (12 pts.) Suppose

$$f(x) = x^4 - 4x^3 + 4x^2 + 18.$$

(a) Use the first derivative to find the critical points of the graph of $y = f(x)$ and determine which are local minima and which are local maximum.

(b) Use the second derivative to find inflection points of the graph of $y = f(x)$.

(c) Determine where the graph of $y = f(x)$ is concave up and concave down.

(d) On the additional paper provided, carefully draw a graph of the curve $y = f(x)$. Label all points found above.

5. (12 pts.) Suppose

$$f(x) = \cos^2 x - 2\sin x \quad (-6 \leq x \leq 6).$$

(a) Use the first derivative to find the critical points of the graph of $y = f(x)$ and determine which are local minima and which are local maximum.

(b) Use the second derivative to find inflection points of the graph of $y = f(x)$.

(c) Determine where the graph of $y = f(x)$ is concave up and concave down.

(d) On the additional paper provided, carefully draw a graph of the curve $y = f(x)$. Label all points found above.

Exam 2, Part B

Calculators are NOT allowed on Exam 2, Part B.
There are 5 problems on 5 pages.
You will be graded on your work and communication of it.

1. (12 pts.; 3 pts. for each part.) Find $\dfrac{dy}{dx}$.

   (a) $\sin y + x^2 y = \pi e^2$.

   (b) $y^3 + xy^2 = \cos x$.

   (c) $y = \ln(x^3 + 5x)$

   (d) $y = \ln(\sec x)$

2. (12 pts.; 6 pts. for each part.)

Compute $\frac{dy}{dx}$. Put your answer in terms of x if possible.

(a) $\ln y = x \ln x$

(b) $\sin y = x$

3. (10 pts.) A girl flies a kite at a height of 50 meters. The wind carries her kite horizontally away from her at a rate of 10 meters per second. How fast must she let out the string when the kite is 130 meters away from her?

4. (4 pts.)

   (a) Find the equation for the line through $(0, 1)$ with slope 1.

   (b) Find the equation for the line through $(4, 2)$ with slope $\frac{1}{4}$.

5. (8 pts.)

   (a) Linearize each of the functions $f(x)$, below, at the point $(a, f(a))$.

      i. $f(x) = e^x$, $a = 0$.

      ii. $f(x) = \sqrt{x}$, $a = 4$.

   (b) Use your work in part (a), above, to estimate the values of:

      i. $e^{0.01}$.

      ii. $\sqrt{3.9}$.

6. (6 pts.) Suppose x is a positive real number.

(a) Find a formula for $\sec(\tan^{-1}(x))$.

(b) Find a relationship between $\tan^{-1}(x)$ and $\cot^{-1}(x)$.

(c) Find a relationship between $\tan^{-1}(x)$ and $\tan^{-1}(1/x)$.

7. (5 pts.) Explain *why* the derivative of $y = \ln x$ is $y' = 1/x$.