Assignment 8

This assignment is due at the beginning of class on Monday, May 5, 2008.

1. (a) For each definite integral below, use $n = 10$ rectangles to approximate the area under the curve. Please give answers to at least four decimal places.

   i. $\displaystyle\int_1^2 \frac{1}{x} dx$

   ii. $\displaystyle\int_1^4 \frac{1}{x} dx$

   iii. $\displaystyle\int_1^8 \frac{1}{x} dx$

   (b) What do you notice about your answers, above? (What is the pattern?) Use the observed pattern to guess at the value of $\displaystyle\int_1^{32} \frac{1}{x} dx$.

   (c) Use the Fundamental Theorem of Calculus to <u>explain</u> the pattern you observed in part (b).

2. We know, from an earlier assignment, that the derivative of $y = \tan^{-1}(x)$ is $y' = \frac{1}{1+x^2}$ and so, according to the Fundamental Theorem of Calculus,

$$\int \frac{1}{1+x^2}\, dx = \tan^{-1}(x) + C.$$

   In this problem we solve the indefinite integral $\displaystyle\int \frac{1}{1+x^2}\, dx$ directly, using a certain substitution. Here is how:

   The expression $1 + x^2$ should remind us of the Pythagorean Theorem. Draw a right triangle with legs of lengths 1 and x. (What is the length of the hypotenuse?) Let θ represent the angle opposite the leg of length x, so that $\tan(\theta) = x$. Now substitute into your integral so that everything is written in terms of θ, not x.

3. Use $n = 10$ rectangles to approximate, as carefully as possible, the definite integral $\displaystyle\int_0^1 \frac{1}{1+x^2}\, dx$. Then explain how your answer provides an approximation for the number π.

Hints and Suggestions for Assignment 8

1. To get the most accurate answer, you should do both the left endpoint estimate and the right endpoint estimate. (Simplify your work when possible.) Then average the two.

   The pattern should be pretty obvious (look for multiples.)

2. Since $\tan(\theta) = x$, we can solve for the differential dx and get an expression in terms of θ. (If $x = \tan(\theta)$, what is dx?) We can also rewrite $\sqrt{1 + x^2}$ in terms of θ (look at the triangle!) and so we can turn the entire integral $\displaystyle\int \frac{1}{1+x^2}\, dx$ into an integral which has no x's in it and in which everything is in terms of the new variable θ.

   Do that. Then evaluate that new integral. Finally, replace the appearances of θ with the appropriate expressions in terms of x.

3. To get the most accurate answer, you should do both the left endpoint estimate and the right endpoint estimate. Then average the two.

   (As a historical interest, a modification of this idea was used in the 1800s to calculate π to over 500 digits. We won't attempt to replicate that here....)