Assignment 7

Due Monday, April 14, 2008

Suggested steps for solving optimization problems:

1. Identify important quantities and draw a diagram or picture.
2. Collect information, writing down equations implied by the problem.
3. Write a single equation involving the quantity to be optimized. Use this to create a formula for this quantity in terms of just <u>one</u> variable.
4. Find the critical points of the function in the previous part.
5. Determine whether these critical points give max or mins. Do you need to check the boundaries of your domain?
6. Write out your answer in English, with proper units. (Does your answer make sense?)

<u>Use</u> these steps in doing the problems, below. You should turn in problems 1-5.

Optimization problems:

0. A farmer has 100 feet of fence and wants to fence in a rectangular pen. What dimensions will maximize the enclosed area?

1. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of length 3 cm. and 4 cm. if two sides of the rectangle lie along the legs of the triangle.

2. We want to make a box with an open top. The length should be twice the width and the box should hold 200 cubic feet. What dimensions will minimize the amount of material needed to construct the box?

3. Find the point on the graph of $y^2 = x^3 + 4$ that is closest to the origin.

4. A steel pipe is carried down a hallway 10 feet wide. There is a right turn into a hall 5 feet wide. What is the length of the longest pipe that can be carried horizontally around the corner?

5. A piece of wire one meter long is to be cut in two pieces. One piece is bent into a square and the other is bent into a circle. Where should the wire be cut to
   (a) maximize the enclosed area?
   (b) minimize the enclosed area?

Hints and Comments on Assignment 7

0. **Solution.** Draw a rectangular pen and label the width and lengths with variables w, l respectively. We know that the area of the pen is $A = lw$ while the amount of fencing is $F = 2l + 2w$. We also know that $F = 100$ feet so that $100 = 2l + 2w$. We want to maximize A but the formula $A = lw$

involves two variables. Use the equation $100 = 2l + 2w$ to solve for l and get $l = 50 - w$. Substitute for l in the equation for A and get

$$A = (50 - w)w = 50w - w^2.$$

Now that we have A in terms of just one variable, we may differentiate and obtain

$$dA/dw = 50 - 2w.$$

The quantity $5 - 2w$ is defined everywhere. It is zero when $w = 25$ so this must give either a minimum or maximum. We note that the second derivative of A is

$$d^2A/dw^2 = -2$$

which is negative so the function is concave down when $w = 25$ and so $w = 25$ must give a maximum for the area. The dimensions then are $l = w = 25$ and so the rectangular pen is a square with sides of length 25 feet.

1. One corner of the rectangle will be at the right angle of the triangle; the opposite corner of the rectangle will be on the hypotenuse of the triangle. You know the area of a rectangle but you must use your understanding of lines to replace one of the variables.

2. What is to be minimized? Surface area.

3. You want to minimize "distance from the origin." The Pythagorean Theorem gives a formula for distance D between a point (x, y) and the origin $(0, 0)$. Unfortunately this involves a square root. Here is a hint to simplify your work: the maximum or minimum of a quantity D occurs at the same place as the maximum/minimum of D^2. So don't try to minimize D; minimize instead the quantity D^2, the "square of the distance from the origin."

4. Draw a picture of the hallway with the largest possible pipe as it is "stuck" going around a corner in the hallway. This should give you a right triangle. Can you write the length L of the pipe in terms of just one variable, θ?

5. When you set up the proper equation, you will discover several critical points. One of these will be the maximum; another will, hopefully, be a minimum?