Assignment 6

Due Monday, April 14, 2008

1. Explain the relationship between limits of functions and the concept of the derivative. (Why do we concern ourselves about limits in a calculus class? If we are taking limits of *functions* when we compute a derivative, what function is involved in that limit?)

2. Let $f(x) = \sin x$ and $a = \pi/6$. In class we agreed that the limit of $f(x)$ as x approaches a is $L = \frac{1}{2}$. That is, we agreed that
$$\lim_{x \to \frac{\pi}{6}} \sin x = \frac{1}{2}.$$
This problem explores the meaning of this statement; throughout this problem $f(x) = \sin x$ and $a = \pi/6$.
   (a) For each ϵ, below, compute the <u>best</u> δ, to four decimal places, such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.
      i. $\epsilon = 0.1$
      ii. $\epsilon = 0.05$
      iii. $\epsilon = 0.01$
   (b) Suppose ϵ is given and *very* small. What will be the "best" choice here for δ in the limit definition? (Note that in this part of the problem, unlike part (a), you are not being given ϵ. Your answer should describe δ in terms of the unknown ϵ.)

3. For this problem, let $f(x) = \frac{x^3-27}{x-3}$ and $a = 3$.
   (a) Compute
$$\lim_{x \to a} f(x).$$
   Explain your answer.
   (b) For each ϵ, below, use the function $f(x)$ given above, with $a = 3$, and compute the <u>best</u> δ, to four decimal places, such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.
      i. $\epsilon = 0.1$
      ii. $\epsilon = 0.05$
      iii. $\epsilon = 0.01$
   (c) Suppose ϵ is given and *very* small. What will be the "best" choice here for δ in the limit definition? Why?

Hints and Comments on Assignment 6

1. Use several paragraphs to explain the definition of the derivative in your own words.

2. In part (a), you must use your graphing calculator. In part (b) you <u>must</u> use the derivative! (Assuming that ϵ is very small and that you get ϵ and δ just right, what does the graph look like in your calculator window? And what is the ratio of ϵ to δ in that case?)

3. Problem 3 is similar to problem 2.