Assignment 3, Spring 2008

This assignment is due Monday, February 25, 2008, at the beginning of class.

1. In each problem below, y is defined implicitly by a certain equation. Use implicit differentiation to find dy/dx. *If possible*, put your final answer in terms of just the variable x.

   (a) $\sin y = x$.

   (b) $\ln y = x \ln x$.

   (c) $x^2 + y^2 = 1$.

   (d) $(x + y)^3 = xy$.

2. We define a certain function $\cosh(t)$ in terms of the variable t as follows:

$$\cosh(t) = \frac{1}{2}(e^t + e^{-t}).$$

   This function is sometimes called the *hyperbolic* cosine of t. The *hyperbolic* sine function is similarly defined by

$$\sinh(t) = \frac{1}{2}(e^t - e^{-t}).$$

   Use these definitions in the following problems.

   (a) By analogy to the ordinary trig functions, develop formulae for the hyperbolic tangent, hyperbolic cotangent, hyperbolic secant and hyperbolic cosecant functions.

   (b) Show that the derivatives of the hyperbolic sine function is the hyperbolic cosine function, and conversely, that the derivative of the hyperbolic cosine is the hyperbolic sine.

   (c) Compute the derivative of the remaining four hyperbolic trig functions.

3. (Continuation of the previous problem.)

   (a) Show that $\cosh(t)^2 - \sinh(t)^2 = 1$. (What is the analogous equation for the ordinary trig functions?)

   (b) Use the ideas in section 3.7 of our textbook (by Thomas, 11th edition) to compute the derivative of the inverse hyperbolic sine function $f(x) = \sinh^{-1}(x)$. Put your final answer in terms of the variable x.

Comments and suggestions for Assignment 6

1. In part (a), after doing the implicit differentiation, you can explicitly solve for x if you can write $\cos x$ in terms of $\sin x$. (Use the Pythagorean Theorem of ordinary trig.)

   In part (b), use the fact that the derivative of $\ln x$ is $1/x$. Also use the product rule. You *can* explicitly solve for y in terms of x

   In part (c), you can, at the end of your work, write the answer in terms of the variable x.

   You can*not* write your answer to part (d) in terms of just the variable x. The variable y will be involved if your solution for dy/dx.

2. Since the ordinary tangent is defined by $\tan(x) = \frac{\sin(x)}{\cos(x)}$, then by analogy we would assume that the hyperbolic tangent of t is $\tanh(t) = \frac{\sinh(t)}{\cosh(t)}$. Now plug in the formulae in the definitions of $\cosh(t)$ and $\sinh(t)$ and simplify in order to get an equation defining $\tanh(t)$ in terms of t. Do similar work for the other hyperbolic trig functions.

3. Part (a) is straightforward: replace $\cosh(t)$ by $\frac{1}{2}(e^t + e^{-t})$ and replace $\sinh(t)$ by $\frac{1}{2}(e^t - e^{-t})$ in the given equation, then square, subtract, simplify.

   Part (b) follows the same line as the derivative of the inverse sine function (done in the book and in the class lecture.)