Assignment 1, MTH 142
Spring 2008

This assignment is due at the beginning of class on **Tuesday, February 5.**

Please make sure to follow the syllabus instructions regarding assignments. Please put each problem on a separate page. The style of your presentation is very important; please write clearly. In addition to clarity, your writing should use correct grammar and spelling.

It is important to *think* about the mathematics you are doing. Please write down any mathematical observations related to the given problems.

1. I am driving from Huntsville to Minute Maid Park in Houston for the Astros home opener. (April 7 vs. St. Louis, just fyi.) I drive slowly from my home near the intersection of Sycamore and Palm Street and then average about 40 miles per hour on Sam Houston Avenue as I head south towards the highway 19 entrance to I-45. On I-45 I speed up to 75 miles an hour for a time but then slow down near the Woodlands as traffic begins to get dense. From then on, the traffic is stop-and-go all the way into Minute Maid Park.

   The game lasts 2 1/2 hours. Afterwards, driving home, I am talking about the game with my friends and I accidentally get on I-10 headed west and drive 5 miles before I finally get turned around and go back to get on I-45 north. After that, traffic is light and I can travel between 65 and 70 most of the way back home to Huntsville.

   (a) Let s be the straight line distance in miles between me and my Huntsville home. Let t be the time measured in hours, with $t = 0$ representing the beginning of my trip. Use the details, above, to draw a graph of s as a function of t.

   (b) Let $v(t) = s'(t)$ be the "velocity", that is, the instantaneous rate of change of s with respect to t. Graph v as a function of t. (Note that $v(t)$ can be negative.)

   (c) Let $a(t) = v'(t)$ be the "acceleration", that is, the instantaneous rate of change of v with respect to t. Graph a as a function of t. (The function $a(t)$ can also be negative.)

2. You are given a series of functions, $f(x)$, below. For each function, estimate the derivative $f'(0)$ in the following way. Use your calculator to compute the slope of secant lines joining $(0, f(0))$ and $(h, f(h))$ for small values of h. Let h get closer and closer to zero and guess the limit as h goes to zero.

   (a) $f(x) = \sin(x)$, where your calculator is set in "degree" mode,

   (b) $f(x) = \sin(x)$, where your calculator is set in "radians" mode,

   (c) $f(x) = 2^x$,

   (d) $f(x) = e^x$,

(e) $f(x) = 3^x$.

3. Consider the following functions

   (a) $f(x) = x^5$,
   (b) $f(x) = \sqrt{x}$
   (c) $f(x) = \frac{1}{x}$

   Use the formal definition of the derivative to compute $f'(x)$ in each case.

   What pattern do you see?

Suggestions and hints for the various problems

1. You have some flexibility in this problem so you want to explain your drawing. You might put each graph on a separate sheet of paper, with notes to describe what is happening where.

2. Try to guess your answers to at least four decimal places!

3. Part (a) is best done using the binomial theorem (or "Pascal's triangle".) If you don't know the binomial theorem, look it up on Wikipedia!

   Parts (b) and (c) require a little bit of algebra.

   Once you notice the pattern, you will be duplicating the work of Newton and Leibniz, from the 1600s, over 350 years ago.