Distributional Effects of Environmental Policy: The Case of Mexico

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PRELIMINARY VERSION

Abstract

This study develops an analytical model with three goods: clean agricultural, clean manufacturing and dirty manufacturing. The production of agricultural goods has land as a fixed factor. The model is calibrated using data for Mexico. The introduction of environmental taxes and a tax cut in the manufacturing goods to keep government revenue unchanged increases welfare but at the cost of a more unequal distribution of income. The costs of the policy are mostly regressive. The regressivity of the policy is driven by tax cut on manufacturing rather than the environmental tax. The income effects from land for Mexico are relatively small.

1 Introduction

Distributional effects are important social concerns, and environmental policy has two kinds of effects on the distribution of real income. First, the distribution of benefits among income groups depends on the values that each group places on abating a particular pollutant. Second, the distribution of the costs of depends on the regulatory instrument used and the way that the program is financed.

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The benefits depend on how much each group values environmental quality. If environmental quality is a luxury good, then high-income groups will have a higher valuation of an improvement in environmental quality than low-income groups. And, according to Harrison (1994), costs also tend to be regressive, and the benefits do not compensate the poor when differences in rents and valuation are taken into account.

This study develops an analytical model with one clean agricultural good, one clean manufacturing good and one one dirty manufacturing good. The model is calibrated using data for Mexico. An environmental tax is implemented while the manufacturing tax is reduced to keep government revenue unchanged. I find that the cost of the policy is regressive. The regressivity is driven by the cut in the manufacturing tax rather than the environmental tax. Income groups with the highest expenditure share on the clean manufacturing good benefit the most. Conversely, groups with high expenditure shares in the dirty good and low shares in the clean manufacturing bear higher costs. The regressivity of the costs is reduced by the income effect from land. The distribution of benefits are part of the analytical model. However, for the numerical part, given the lack of reliable data, I abstract from the distribution of benefits across income groups.

Studying the distributional effects of environmental policy is important for policy-makers contemplating the possible implementation of such programs. But most applied studies are based on cost/benefit analysis, that does not say anything about the distributive allocation of resources. Even if the total social benefits of an environmental policy are higher than the total costs, this net gain may not hold for all the different income groups. Moreover, the policy maker ignores if the costs have been fairly distributed. Tietenberg (1988) points out the importance of considering the distribution of the cost and benefits of environmental policy for "ethical" and "pragmatic" reasons.

Interest in the effects of green taxes on income distribution is not new. Much of the research in this area was done in the 70's and 80's. During these two decades, research concentrated on specific programs such as water pollution or air pollution. In the 90's, climate change and greenhouse gases abatement dominates the distributional studies of environmental policy.

In general, these studies are empirical, and they confirm the following:

- Without any kind of compensation to low-income groups, environmental taxes are regressive.

- The regressivity is reduced when income is measured by annual expenditures or by some other measure of lifetime income.

Dorfman(1975) calculates the distribution of the costs of environmental programs among income classes in the U.S. for 1972, and he projects the effects for 1976 and 1980. He finds that the distribution of costs is mildly regressive for 1972 and 1976 and strongly regressive for 1980. By 1980, the regressivity of the environmental program exceeds that of a general sales tax.

Gianessi (1980) studies the impact of the implementation of the Clean Water Act in its 1972 version. This work focus on the burden of taxes and changes in prices of the goods consumed by each income class. He concludes that subsidies may be necessary to alleviate the regressive burden resulting because of the polluters' ability to pass costs to consumers and because of the regressive state and local tax structure.

Robinson (1985) measures the distributional impact of industrial pollution abatement for 1973 and 1977. A main assumption is that the cost of abatement is passed on to consumers in the form of higher prices. A step forward in relation to the previous work is that the author distinguishes between goods consumed by each income class. This study is important because each good has a different cost of abatement. Since the share of expenditure on services at high-income levels on services is large, and since services have low pollution and therefore abatement costs, it is assumed that industries with high abatement cost represent a high share of the lower-income groups' expenditure. The main result is that the distribution of abatement cost burdens across income classes is fairly regressive.

Poterba (1991) uses a partial equilibrium model to confirm that a carbon tax is regressive. He assumes a carbon tax of $100/ton in 1990 and calculates the increase in prices in the energy sector. Then, using data on the share of income and expenditure on energy in each income decile, he obtains the total burden of the tax. The results clearly show that the tax is regressive. As the author points out, however, the use of a partial equilibrium model does not completely describe the distributional effects of a carbon tax. In order to reduce the tax burden on low-income groups, he proposes transfer programs indexed to price changes, increased use of redistributive income taxes, and tax credits for energy expenditures. But, as he states, "*...none of these redistributive schemes will completely offset the distributional impacts of a carbon tax*" (Poterba,1991, pp. 83).

Pearson and Smith (1991), study distributional effects across income groups for the United Kingdom, Ireland, Germany, Italy, Netherlands, Spain and France. The study is more detailed for the United Kingdom, where carbon taxes increase payments for domestic energy and automotive fuels, and the carbon tax is regressive. Assuming fixed behavior (i.e. no response to prices), the regressivity is larger than when households are allowed to respond to higher prices. For the other European countries, the degree of regressivity is much lower (except Ireland), and in the case of Italy the carbon tax is progressive.

Regarding developing countries, Shah and Larsen (1992) estimate the impact of a carbon tax in Pakistan. The authors use different values for the percentage of the tax that is passed on to consumers. They find that, for the intermediate case (where 31% of the carbon tax is shifted toward consumers), the tax is proportional to income and progressive with respect to expenditures.

Jorgenson, Slesnick and Wilcoxen (1992) present a dynamic general equilibrium model using 35 industrial sectors and 16,000 different types of households according to demographic characteristics and wealth. In this extensive work, the authors show that a carbon tax policy that stabilizes emissions to 1990 levels "*is either mildly progressive or mildly regressive depending on the degree of inequality aversion and the measure of progression used*" (Jorgenson, Slesnick and Wilcoxen (1992) pp. 431).

Hamilton and Cameron (1994) calculate the distributional effect of a tax per tonne of CO_2 that is $ 27.7 Canadian Dollars . The authors use a general equilibrium model to solve the problem and find that the tax would be slightly regressive.

In a more recent work, Metcalf (1999) elaborates an empirical study focusing on distributional effects across three types of categories: annual income, lifetime income and cohort analysis. He also considers 40 industries and ten income groups (deciles). The environmental tax increases the prices of goods consumed by the families in each group. In addition, Metcalf (1999) provides calculations of the impact of the environmental tax on prices. The environmental policies considered are taxes on carbon, gasoline, air pollution, and virgin materials. Finally, he concludes that a tax reform equal to ten percent of federal receipts has a small impact on income distribution when the funds are rebated to households through reductions in the payroll and income tax. This provides a basis for feasible environmental reforms.

Walls and Hanson (1999) estimate the costs of a vehicle emission fee as well as costs of the current inspection and maintenance program for each income decile in California. When current income is used as a measure of annual income, they find both policies to be regressive. However, the former is more regressive than the latter. When lifetime income is used, both policies are slightly regressive and similar in their impact.

West and Williams (2002), analyze the incidence of a gasoline tax using an AIDS under different assumptions about how the revenue is recycled. They find that an increase in the gasoline tax is regressive. However, if lump-sum transfers are provided with the additional revenue, the progressivity of the transfer slightly outweighs the regressivity of the tax increase.

Table 1, summarizes the results, model used, policy analyzed and country of the previous literature.

2 The Model

The goal of this paper is to analyze the distributional effects of environmental policy. The analytical model considers two types of distributional effects. First, the change in the aggregate welfare and the welfare change for each income group. Second, the change in the real income of each income group. In the numerical section of the model, I abstract from the distribution of benefits across income groups and consider only the change in aggregate welfare. In particular this study analyzes the distributional effects of environmental policy with pre-existing taxes, a sector specific input (land) in the production of agricultural good and inelastically supplied resources. The model has three goods: agricultural (X), dirty manufacturing (Y) and clean manufacturing (Z). The government taxes only the manufactured goods for distributional considerations. That is, the government assumes that a tax on X will be regressive, as low-income

Table 1: Model Choice, Policy and Result per Country

Author(s)	**Model**	**Policy Analyzed**	**Result**	**Country**
Dorfman (1975)	PE	Environmental Programs	Regressive	USA
Gianessi (1980)	PE	Clean Water Act	Regressive	USA
Robinson (1985)	PE	Industrial Pollution	Regressive	USA
Poterba (1991)	PE	Carbon Tax	Regressive	USA
Pearson and Smith (1991)	PE	Carbon Tax	Varies Across Countries	Europe
Shah and Larsen (1992)	PE	Carbon Tax	Proportional Progressive	Pakistan
Jorgenson et.al (1992)	CGE	Carbon Tax	Mildly Regressive and Progressive	USA
Hamilton and Cameron (1994)	PE	Carbon Tax	Slightly Regressive	Canada
Pench (1998)	CGE	Oil/Consumer Tax	Regressive	Italy
Metcalf (1999)	I-O	Environmental Taxes	Slightly Regressive	USA
Walls and Hanson (1999)	PE	Vehicle Fee Emissions	Regressive	USA
West and Willians (2002)	PE	Gasoline Tax	Regressive, Slightly Progressive	USA

groups spend a higher income share on this good. Such a policy creates two types of effects. First, it produces a an inefficient allocation of resources, as for efficiency considerations all goods should be taxed at the same rate. Second, if land ownership is not evenly distributed across income groups, this policy can create unexpected distributional effects by changing the price of land. In this context, the government introduces a revenue neutral environmental tax. That is, a tax in pollution is imposed while the tax in both manufacturing sectors (dirty and clean) are reduced to keep government revenue the same.

It is important to stress that the model has no income taxes.[1] By eliminating this tax I can focus the analysis on the effect of different taxes in the clean agriculture, clean manufacturing and dirty manufacturing.[2] Thus, I can isolate the effect of environmental taxes across the different income groups. Moreover, one could consider consumption and environmental taxes as additional taxes. Another reason is more practical: if I introduce income taxes then for distributional purposes one would like to have different marginal and average rates across income groups. This would be untractable in this simple analytical model. Finally, for informational and administrative reasons the model has no individual-specific lump-sum transfers.[3]

I solve the model by using the log-linearization method. This method has been previously used given the difficulty of obtaining closed-form solutions and because in some cases it does not require specification of any functional form. Goulder, Parry and Burtraw (1997) use this method to compare the net welfare effects of a policy with revenue recycling effects against a policy with no revenue-recycling effect. Fullerton and Metcalf (2001) use log-linearization to show the net welfare effects of environmental taxes with pre-existing distorting labor taxes. In an extensive paper, Bovenberg and Goulder (2002) review the literature on environmental taxation and second-best analysis. In one part of this work, they consider distributional issues by accounting for

[1] If resources are inelastically supplied for optimality reasons, all government revenue requirement is obtained from taxing resources.

[2] See Fullerton (1997), Bovenberg and De Mooij (1997a) and De Mooij (2000) for a discussion on the normalization of the tax rates.

[3] If I had individual specific lump-sum transfers then the government could offset any distributional effect without creating any distortion.

two different types of households using the log-linearization approach. However, the focus is on the trade-off between wages and increasing environmental quality. That is, they study the effect of income distribution on environmental quality and not the effects of environmental policies on income distribution.

2.1 Consumption

The model divides consumers into ten different types, one for each income decile. Agents obtain income from the rent of agricultural land, A, and all other resources, R. These resources include labor, capital and other resources used in production with the exception of agricultural land. I assume that resources are inelastically supplied.[4]. Each agent obtains utility from the consumption of agricultural goods (X), dirty manufacturing goods (Y), clean manufacturing goods (Z), public goods supplied by the government (G) and environmental quality (E). The problem of the representative consumer in the income group h is the following

$$\max_{X^h,Y^h,Z^h} \quad U(X^h, Y^h, Z^h; G, E)$$

Subject to

$$R^h + P_A A^h = P_X X^h + (P_Y + t)Y^h + (P_Z + t)Z^h \tag{2.1}$$

The demand function from the previous problem is the following

$$Q_i^h = Q_i^h(R^h, A^h, P_X, P_Y, P_Z, t) \qquad \text{for} \quad i = X, Y, Z \tag{2.2}$$

The price of the resources (P_R) is the numeraire and consumption tax rates are the same for all manufacturing goods. Moreover, new emissions have a negative effect on the environment. This effect is captured by the following function: $E = e(W)$, where W is the total amount of waste emissions. Hence, environmental quality depends on the waste produced by the industries.

[4] This provides another reason to ignore the income tax, since it would be equivalent to a lump sum tax

2.2 Aggregation of consumers

As can be seen from Eq. (2.2) each income-group has its own demand function for X, Y and Z. In order to obtain a representative consumer demand for each good I use an aggregate demand function and then disaggregate for each income group. This is a problem of exact aggregation. A utility function, and in particular the share of consumption on each good, must satisfy certain conditions for exact aggregation to hold (see Deaton and Muellbauer ,1982,).[5] If these conditions hold, then the consumer demand for each good can be written as:

$$Q_i = f_i(R, A, P_X, P_Y, P_Z, t) = \sum_h Q_i^h(\quad R^h, A^h, P_X, P_Y, P_Z, t) \qquad \text{for} \quad i = X, Y, Z \tag{2.3}$$

where

$$R = \sum_h R^h \qquad A = \sum_h A^h \qquad Q_i = \sum_h Q_i^h$$

Note that I have not imposed neither identical preferences nor homotheticity.

2.3 Production and Government

The production of X, Y, Z and W use the following production functions, respectively:

$$X = X(R_X, A_X) \tag{2.4}$$

$$Y = Y(R_Y, W) \tag{2.5}$$

$$Z = R_Z \tag{2.6}$$

$$W = R_W \tag{2.7}$$

where R_i is the amount of resources used in the production of the i good (for $i = X, Y, Z$). Note that emissions require some private costs for removal and disposal; the resources used for this purposes are denoted by R_W. In the same way as Fullerton and Metcalf (2001), Eq. 2.7 implies that the private cost of W is equal to $(P_W + t_W)$. Given the normalization of prices and the fact that the original tax on pollution is zero, the private cost of W is one. One could argue that all manufacturing goods are polluting.

[5] These conditions are derived in Appendix A.

Nevertheless, setting energy and fuels as the polluting goods is a close approximation given that such sectors are the heavy polluters. The amount of agricultural land used in the production of the clean good is A_X. Land is a fixed input in the production of X, however each firm considers it as non-fixed. An important piece of information is given by the elasticity of substitution of inputs in the production of X and Y.

$$\sigma_X = \frac{d[\frac{A}{R_X}]/\frac{A}{R_X}}{d[\frac{P_A}{P_R}]/\frac{P_A}{P_R}} \tag{2.8}$$

$$\sigma_y = \frac{d[\frac{R_Y}{W}]/\frac{R_Y}{W}}{d[\frac{P_R}{P_W}]/\frac{P_R}{P_W}} \tag{2.9}$$

Note that from Equations (2.8) and (2.9) I implicitly define the elasticities of substitution as negative. Next, I present the economy resource constraints:

$$\begin{aligned} R &= R_X + R_Y + R_Z + R_W + R_G \qquad (2.10) \\ A &= A_X \end{aligned}$$

The government obtains revenue from a tax on both manufacturing goods and a tax on pollution. The revenue is used to provide a public good, G. In order to provide a fixed amount of this public good, the government uses R_G of resources, $G = R_G$. The government budget constraint is the following

$$G = (Y + Z)t + Wt_W \tag{2.11}$$

Finally, in a competitive equilibrium with constant returns to scale, profits in all industries are zero. The zero profit conditions for each industry are the following

$$P_X X = P_R R_X + P_A A \tag{2.12}$$

$$P_Y Y = P_R R_Y + (P_W + t_W)W \tag{2.13}$$

$$P_Z Z = P_R R_Z \tag{2.14}$$

The pollution tax affects two equations directly: one, the government budget constraint and, two, the zero profit condition of Y.

2.4 Log-linearization

The next step is to transform these equations into percentage change using the log-linearization method. I linearize Equations (2.3) to (2.14) and denote the percentage change in each variable with a hat (e.g. $\widehat{X} = \frac{dX}{X}$, with exception of the change in the tax that is defined as: $\widehat{t} = \frac{dt}{1+t}$).[6] By inspecting Equations (2.6) and (2.14) I know that $\widehat{P}_R = \widehat{P}_Z$. Moreover, since P_R is the numeraire, $\widehat{P}_R = \widehat{P}_Z = 0$. Table 2 shows the equations in deviation form and the parameters of the model.

Note that I have already set $\widehat{G}$, $\widehat{R}$ and $\widehat{A}$ equal to zero. The model has eleven equations in deviation form and ten endogenous variables in percentage change form. The endogenous variables are: $\widehat{P}_Y, \widehat{P}_X, \widehat{Y}, \widehat{X}, \widehat{W}, \widehat{Z}, \widehat{R}_X, \widehat{R}_Y, \widehat{t}$ and $\widehat{P}_A$. The eleven equations are numbered from Eq. (2.19) to (2.15). Note that I have more equations that unknowns, yet, by Walras' Law I can discard Eq. 2.22 and the numeraire is the price of resources, $P_R = 1$. Thus, the linear system is reduced to ten equations and ten unknowns.

2.4.1 Welfare

The effects on welfare are calculated as the marginal excess burden (*meb*) as in Keller (1980) and De Mooij (2000). For marginal changes the marginal excess burden is equal to the compensating and equivalent variation. By definition if the *meb* is positive, then the policy reduces welfare, as the individual needs to be compensated to stay in the same level of utility. The *meb* for the representative consumer in the income-group h is the following,

$$meb^h = -\psi_A^h \widehat{P}_A + \widehat{t}(1 - \psi_X^h) + \psi_X^h \widehat{P}_X + \gamma \psi_Y^h \widehat{P}_Y + \psi_W^h \varrho^h \widehat{W} \tag{2.26}$$

where

$$\begin{aligned} \psi_A^h &= \frac{A^h P_A}{P_R R^h + P_A A^h} \\ \psi_i^h &= \frac{(1+t) Q_i^h}{P_R R^h + P_A A^h} \\ \psi_W^h &= \frac{W P_w}{P_R R^h + P_A A^h} \\ \varrho^h &= \left(\frac{\partial U^h}{\partial E} \frac{\partial E}{\partial W} \right) \frac{1}{\lambda^h} \end{aligned}$$

The term ϱ^h represents the marginal environmental damage in monetary terms. A negative value of meb^h indicates an increase in the welfare of the income group h. Given

[6] The derivations of the equations are shown in Appendix B.

Table 2: Linearized Model

$\widehat{X} = \widehat{P}_X \varepsilon_X + (\gamma \widehat{P}_Y + \widehat{t})\varepsilon_{XY} + \varepsilon_{XZ}\widehat{t} + \varepsilon_{XP_A}\widehat{P}_A$	Demand for X	(2.15)
$\widehat{Y} = (\gamma \widehat{P}_Y + \widehat{t})\varepsilon_Y + \varepsilon_{YX}\widehat{P}_X + \varepsilon_{YZ}\widehat{t} + \varepsilon_{YP_A}\widehat{P}_A$	Demand for Y	(2.16)
$\widehat{Z} = \widehat{t}\varepsilon_Z + \varepsilon_{ZX}\widehat{P}_X + \varepsilon_{ZY}(\gamma \widehat{P}_Y + \widehat{t}) + \varepsilon_{ZP_A}\widehat{P}_A$	Demand for Z	(2.17)
$\widehat{X} = \widehat{R}_X \theta$	Prod. Function of X	(2.18)
$\widehat{Y} = \widehat{R}_Y \phi + \widehat{W}(1-\phi)$	Prod. Function of Y	(2.19)
$\widehat{R}_X = -\sigma_X \widehat{P}_A$	Subst. in Prod. of X	(2.20)
$\widehat{R}_Y - \widehat{W} = -\sigma_Y \widehat{t}_W$	Subst. in Prod. of Y	(2.21)
$0 = \widehat{R}_X \mu_X + \widehat{R}_Y \mu_Y + \widehat{Z}\mu_Z + \widehat{R}_W \mu_W$	Resouce Constraint	(2.22)
$0 = \psi_X \widehat{t} + \psi_Z \widehat{t} + \delta\psi_Y \widehat{Y} + \delta\psi_Z \widehat{Z} + \psi_W \widehat{t}_W$	Gov. Budget Constraint	(2.23)
$\widehat{P}_X + \widehat{X} = \theta \widehat{R}_X + (1-\theta)\widehat{P}_A$	Zero Profits in X	(2.24)
$\widehat{Y} + \widehat{P}_Y = \phi \widehat{R}_Y + (1-\phi)(\widehat{W} + \widehat{t}_W)$	Zero Profits in Y	(2.25)

Elasticities

ε_i	uncompensated price elasticity of $i \quad \forall \quad i = X, Y, Z$
ε_{ij}	uncompensated cross-price elasticity $\forall \quad i \neq j, \quad i, j = X, Y, Z$
ε_{iP_A}	elasticity of i to $P_A \quad \forall \quad i = X, Y, Z$

Shares and Parameters

$\phi = \frac{R_Y}{Y}$	share of resources to production in Y
$\theta = \frac{R_X}{X}$	share of resources to production in X
$\mu_i = \frac{R_i}{R}$	share of resources in i to total resources
$\psi_i = \frac{(1+t)Q_i}{P_R R + P_A A}$	share of expenditure of i in total expenditure
$\gamma = \frac{1}{1+t} \quad \delta = \frac{t}{1+t}$	parameters

that the government revenue does not change, Eq. 2.26 represents the excess burden only from the inefficient collection of revenue. The effect in welfare for each income group can be divided in three parts. The first part is the change in income. This effect is given by the first term in the right hand side of Eq. (2.26). The second term is the change in the consumer prices, given by the second to fourth terms. The third effect, given by the last term, is the change in emissions and the marginal environmental damage.

The differences in meb^h across income groups are caused by differences in their share of expenditure, income from land and their respective marginal environmental damages. Note that the first term in the right hand side of Eq. (2.26) represents the increase in land income for the h group. The income groups where land income represents a higher share of their total income will benefit more from an increase in the price of land. The second to fourth terms represent the change in the value of expenditures. Note how the change in prices and taxes is weighted by the relative share of expenditures of each good.

The aggregate marginal excess burden ($ameb$) is obtained in the same way as meb^h but using the aggregate utility and income. The $ameb$ represents the overall welfare effect of the policy and it is simplified to the following expression

$$ameb = -\psi_A \widehat{P}_A + \widehat{t}(1 - \psi_X) + \psi_X \widehat{P}_X + \gamma \psi_Y \widehat{P}_Y + \psi_W \widehat{W} \sum_h \varrho^h \tag{2.27}$$

Combining (2.26) and (2.27) I can see that the aggregate marginal excess burden is equal to the weighted average of the marginal excess burden of all income groups. These weights are equal to the shares of each income group in total income.[7] Thus, the high-income groups have a relative higher weight in the $ameb$. A negative value of $ameb$ indicates that the overall welfare increases.

Given the lack of reliable information, I do not consider the distribution of benefits across income groups and concentrate in the distribution of the costs. One could argue that benefits are mostly progressive since low-income groups tend to have higher exposures to contaminants.[8] However, in this model, I care about the valuation of the

[7] That is, $ameb = \sum_h \frac{P_R R^h P_A A^h}{P_R R P_A A} meb^h$

[8] See Harrison and Rubinfeld (1978).

damages and one can expect high-income groups to place a higher valuation on the environment. In the case where the marginal environmental damages are considered the same across income groups, the last term of Eq. (2.27) would be: $\psi_W \widehat{W} H \varrho$. This would imply that the marginal utility of income, λ^h, is proportional to the lost utility from a marginal increase in emissions. As De Mooij (2000) pointed out, if high-income groups have a lower marginal utility of income, then in order to keep ϱ^h constant they must also have a lower proportional valuation of the environment in utility terms.

2.5 Tax on emissions

An increase in environmental taxes changes the government budget constraint and the zero profit condition for Y. The introduction of environmental taxes will reduce the tax on the manufacturing goods as long as the demand for pollution is on the left side of the Laffer curve and the demand for Y increases.[9] The producer price of Y increases because the price of pollution increases by the tax change and pollution is an input of Y. Hence, if the tax on manufacturing decreases, the effect on the consumer price of Y is indeterminate. In addition, the reduction in emissions reduces the amount of resources R_W used in this sector and the output of Y falls.

As a result of the normalization the producer price of Z does not change ($\widehat{P}_Z = 0$). Thus, the reduction of the manufacturing tax reduces the consumer price of Z increasing the quantity demanded of Z. The higher output of Z increases the demand for resources in this sector.

Depending on whether X and Z are gross substitutes or gross complements, the increase in the consumer price of Z may increase or decrease the demand for X. If X and Z are gross substitutes then the demand for X and A_X decreases. Depending on the whether X and Y are gross substitutes or gross complements, the increase in the consumer price of Y may increase or decrease the demand for X. If the goods are gross complements then the demand for X goes up. If the release of resources from Y is not enough to compensate for the higher resource demand from Z, the amount of resources used in X falls. If R_X drops then agricultural production falls. Also, less use

[9]If the demand for pollution is on the left side of the Laffer curve an increase in the pollution tax increases the tax revenue from pollution.

of resources implies a lower demand for land and its price falls. This also produces a drop in the price of X.

The change in relative prices is likely to have different effects on each income group. Assuming the consumer price of Y increases, then income groups with higher expenditure shares in Y are the most affected. Similarly, the lower consumer price of Z benefits income groups with high expenditure shares in Z.

The trade off between efficiency and equity from the environmental tax can be analyzed using Equations (2.26) and (2.27). If the wealthiest income groups have higher expenditure share on the clean manufacturing and low expenditure shares in the dirty manufacturing, then the environmental tax might increase efficiency but at the cost of redistributing income to wealthiest groups. Assuming that land rents shares are higher for the wealthiest groups, the distributional effects can be exacerbated or reduced depending on whether the price of land increases or decreases. If P_A increases then, the environmental policy may create scarcity rents from land mainly captured by the high-income groups. Moreover, it increases the price of X, that represents a big expenditure share of low-income groups.

3 Analytical Results

I start by obtaining the prices and tax response to the pollution tax: $\widehat{P}_X, \widehat{P}_Y$, $\widehat{P}_A$ and $\widehat{t}$. Substituting Eq. (2.19) into (2.25) I obtain the value for the percentage change in the price of Y.

$$\widehat{P}_Y = (1 - \phi)\widehat{t}_W \tag{3.1}$$

Eq. (3.1) shows the proportional change in the price of the dirty good as a response to a percentage change in the tax on pollution. The price change depends on the share of pollution (as an input) in the total output of the dirty good. If pollution represents a big share in the total inputs of Y, then (1 - ϕ) will be high and the price change in Y will also be higher. Note that what is important is not the level of emissions, but the level of emissions with respect to output. The result in Eq. (3.1) holds only if the original tax on pollution is zero.

The next step is to obtain an expression for $\widehat{P}_X$ and $\widehat{P}_A$. Using Equations (2.18) and (2.24), I obtain the following equation that relates the price of land to the price of the agricultural goods.

$$\widehat{P}_X = (1-\theta)\widehat{P}_A \tag{3.2}$$

From Eq. (3.2) I can see that the price changes in the agricultural sector and the price change of land are proportional. The rate at which the price of food and beverages goes up depends on the share of land in the total output of agricultural goods. Since the price of resources is fixed, the change in the price of X is a result of the change in the price of land.

Solving the system of equations given by (2.19) - (2.15) I obtain an expression for the necessary tax change in manufacturing to balance the government budget constraint as a function of the exogenous environmental tax change.

$$\widehat{t} = \left\{ \frac{\gamma(1-\theta)(\Phi_1\varepsilon_{XY} - \frac{\psi_Z}{\psi_Y}\varepsilon_{ZY} - \varepsilon_Y) - \frac{\psi_W}{\psi_Y}}{-\Phi_1(\varepsilon_{XY}+\varepsilon_{XZ}) + (\varepsilon_{ZY}+\varepsilon_Z)\frac{\psi_Z}{\psi_Y} + \varepsilon_Y + \varepsilon_{YZ} + \frac{\psi_Y+\psi_Z}{\delta\psi_Y}} \right\} \widehat{t}_W \equiv \zeta\widehat{t}_W \tag{3.3}$$

where

$$\Phi_1 = \frac{-\psi_Z(\varepsilon_{ZX}(1-\theta)+\varepsilon_{ZP_A}) + \psi_Y(\varepsilon_{YX}(1-\theta)+\varepsilon_{YP_A})}{-\psi_Y(\theta\sigma_X + \varepsilon_{XP_A} + (1-\theta)\varepsilon_X)} \tag{3.4}$$

The change in the manufacturing taxes depends on the elasticities and the shares. It is necessary to use specific values of the parameters and elasticities to sign ζ. If the increase in the pollution tax increases government revenue then the tax in manufactures will decrease. In this case, ζ will be negative.

Next, I will analyze the effect on the price of X and A. Combining Equations (2.18) and (2.24) I obtain the following expression.

$$\widehat{P}_X = (1-\theta)\left[\frac{\varepsilon_{XY}\gamma(1-\phi)\widehat{t}_W + \widehat{t}[\varepsilon_{XY}\varepsilon_{XZ}]}{-\theta\sigma_X - \varepsilon_{XP_A} - (1-\theta)\varepsilon_X}\right] \tag{3.5}$$

Eq. (3.5) shows that the change in the price of X depends on the different taxes and the own and cross-price elasticities. The first term in the numerator relates the change in pollution and producer price of Y to the change in demand for X. The second term in the numerator relates the change in the tax of manufactures to the change in the demand for Y and Z and to the change in the demand for X. The denominator, β, is the change in the demand for X as response to its own price, the land rent elasticity and the elasticity of substitution in the production of X. Substituting Eq. (3.3) into (3.5) and rearranging terms, I obtain the final expression for the price change of X.

$$\widehat{P}_A = \left[\frac{\varepsilon_{XY}(\gamma(1-\phi)+\zeta)+\varepsilon_{XZ}\zeta}{-\theta\sigma_X - \varepsilon_{XP_A} - (1-\theta)\varepsilon_X}\right]\widehat{t}_W \equiv \pi\widehat{t}_W \tag{3.6}$$

$$\widehat{P}_X = (1-\theta)\left[\frac{\varepsilon_{XY}(\gamma(1-\phi)+\zeta)+\varepsilon_{XZ}\zeta}{-\theta\sigma_X - \varepsilon_{XP_A} - (1-\theta)\varepsilon_X}\right]\widehat{t}_W = \pi(1-\theta)\widehat{t}_W \tag{3.7}$$

Eq. (3.7) shows the price change in the agricultural sector for a given tax on pollution. Note that the effect on $\widehat{P}_X$ will be higher when the share of land on the total output of agriculture is larger. In addition, $\widehat{P}_x$ is not necessarily positive for an increase in t_W, the sign of the expression between the bracket is indeterminate. If the quantity demanded of X goes up, then P_X increases. In this case, ζ would be positive. From Eq. (3.6) I can see that the percentage increase in the price of land is always bigger than the percentage increase in the price of agricultural goods. This is a result of $(1-\theta)$ being a fraction between zero and one. And from Eq. 3.2 I know that the price change in X is equal to the price change in land times a number between zero and one. The increase in the price of land affects the total price of X to the extent that land is used in total production, which is denoted by the term $(1-\theta)$.

Finally, substituting expressions (3.1), (3.3), (3.6) and (3.7) into the meb^h I obtain change in welfare for each income group as a function of pollution tax and the change in pollution.

$$meb^h = \left\{\pi\psi_A^h + \pi\psi_X^h(1-\theta) + \zeta(1-\psi_X^h) + \gamma\psi_Y^h(1-\phi)\right\}\widehat{t}_W + \psi_W^h\varrho^h\widehat{W} \tag{3.8}$$

Admittedly, Eq. (3.8) is a function of an endogenous variable $(\widehat{W})$. However, such an expression allows us to isolate the three effects on welfare. The first term inside the bracket represents the effect on welfare given by the change in income. The second to fourth terms inside the brackets represent the change in consumer prices. The last term of Eq. (3.8) represents the welfare effect from the improvement in the environment. If the expression inside the brackets and the last term are positive then the pollution tax produces a double dividend, -i.e. an increase in real income and better environmental quality.

If the meb^h is negative then welfare goes up. The first and second terms inside the brackets in the right hand side of Eq. (3.8) shows an ambiguous increase in welfare as a result of agricultural goods and land rents moving in the same direction. The third term inside the bracket shows the effect of lower manufacturing taxes on welfare. If such taxes decrease then ζ is negative and welfare increases by the expenditure share of all manufacturing goods on total expenditure. Hence, higher expenditure shares in manufacturing goods makes this term more negative increasing real income. The last term inside the brackets is unambiguously positive; the tax on pollution increases the producer price of the dirty good. Therefore, a higher the share of expenditure in the dirty good tends to make this term more positive and reduce welfare.

From the previous analysis I can conclude that income groups with small expenditure shares in agricultural goods and high expenditure shares in the clean good tend to bear lower costs.

The second term in Eq. (3.8) is the environmental dividend and represents the dollar value of the benefits from a better environment. This term is negative as long as each income group has a positive valuation of the environment and pollution falls. Note that the high-income groups have a lower ψ_W since their income is higher and W is constant across income groups. This tends to increase the benefits of the low-income groups.

The aggregate welfare equation shows the effect of the policy in the overall welfare.

$$ameb = \Big\{\pi\psi_A + \pi\psi_X(1-\theta) + \zeta(1-\psi_X) + \gamma\psi_Y(1-\phi)\Big\}\widehat{t}_W + \psi_W \varrho \widehat{W} \tag{3.9}$$

I can observe the effect of income, price and the environmental quality on welfare as in Eq. (3.8). If both terms on the right hand side of Eq. (3.9) are negative then I have a double dividend for the overall economy. Note that, welfare increases when the *ameb* is negative. In the case where the real income of the lower income groups decreases and the *ameb* is negative, efficiency is improved but at a cost of a more unequal distribution of income.

Without any distributional concerns, the government will set the pollution tax equal to the social marginal environmental damages and all three goods will be taxed at the same rate for efficiency reasons. However, in this model the government does not tax agricultural goods because it is considered to be regressive.

4 Calibration and Data

4.1 Linear Expenditure System

In order to calibrate the model, I assume a specific utility function. As mentioned before, I assume that people are different only in their endowment of resources and land. Low and high-income individuals have the same preferences. They buy different consumption bundles only because their incomes are different. I also want a utility function that does not impose homotheticity, as the data show that expenditure shares across income groups change with income. A utility function compatible with this formulation is the Linear Expenditure System (LES), separable in the consumption of G and E. An agent in the income group h will face the following problem

$$\max_{X^h,Y^h,Z^h} (X^h - \bar{X})^{\alpha_X^h}(Y^h - \bar{Y})^{\alpha_Y^h}(Z^h - \bar{Z})^{\alpha_Z^h} + V(G,E)$$

Subject to

$$R^h + P_A A^h = P_X X^h + (P_Y + t)Y^h + (P_Z + t)Z^h \tag{4.1}$$

The parameter $\bar{Q}_i$ is often referred as "minimum consumption" of good i and α_i as the "marginal expenditure share". This interpretation can be misleading, as $\bar{Q}_i$ can take negative values. In this paper, I consider them only as parameters that help characterize the preferences of each individual.

Next, I show the conditions for exact aggregation for the LES. Setting $IN^h \equiv R^h + P_A A^h$, the demand Eq. (2.2) in section 2.1 can be written as follows

$$P_i^c Q_i^h \quad = \quad P_i^c \bar{Q}_i + \alpha_i^h (IN^h - \sum_i P_i^c \bar{Q}_i) \qquad \forall \quad i = X, Y, Z \tag{4.2}$$

where I normalize such that $\sum_i \alpha_i^h = 1$ for all h. Hence, the aggregate demand in market i is the following

$$\sum_h P_i^c Q_i^h = \sum_h P_i^c \bar{Q}_i + \sum_h \alpha_i^h (IN^h - \sum_i P_i^c \bar{Q}_i) \tag{4.3}$$

Define

$$\sum_h Q_i^h = Q_i \qquad \sum_h \bar{Q}_i = \bar{Q}_i^a \qquad \text{and} \qquad \sum_h IN^h = IN \tag{4.4}$$

where Q_i is the aggregate demand, $\bar{Q}_i^a$ is the aggregate minimum expenditure parameter and IN is the national income. Hence, the condition for exact aggregation is

$$\alpha_i^h = \alpha_i^{h+1} = \cdots = \alpha_i \qquad \forall \quad h = 1, \cdots, H. \qquad \text{and} \qquad i = X, Y, Z \tag{4.5}$$

Thus, the aggregate demand can be written as

$$P_i^c Q_i = P_i^c \bar{Q}_i + \alpha_i (IN - \sum_i P_i^c \bar{Q}_i) \tag{4.6}$$

This formulation allows me to recover the individual demands from the aggregate demand function. Furthermore, the exact aggregation condition and the normalization imply that $\sum_i \alpha_i = 1$.

The advantage of using the LES comes at a cost of imposing certain restrictions. First, the LES assumes that all goods are substitutes. I believe this is not a serious problem in the present model. Second, the price and income elasticities are approximately proportional.

4.2 Data

The data used for this paper is from 2000. First I match our goods used in the model with the categories from the Income Expenditure Survey 2000 for Mexico.

Table 3: Correspondences between the model goods and the IES categories

Goods	Categories
X	Food at home and away
Y	Gasoline, Electricity, Other Fuels 20% of Public Ground Transportation
Z	All other goods and services 80% of Public Transportation

The dirty good Y can be considered as a composite energy good. It includes the production of goods that are considered to be heavy pollutants. I added to this category the amount of gasoline and fuels used in public ground transportation. This is necessary given that the expenditure shares in public transportation are different to those of Z. An environmental tax increases the price of public transportation only by the share of gasoline in output. This share was obtained from Economic Census for Transportation from INEGI (1999). The census publishes the expenditure in gasoline from ground transportation and the total value of output for that industry in 1998. The value of the gasoline share is 0.20. Finally, I imputed the value of gasoline from Z.

I obtain total expenditures by income group in two steps. First, I obtain the total Gross Domestic Product (GDP) for Mexico in 2000 from the INEGI (2003). Second, I obtain the expenditure shares for each income decile and the overall economy from the IES and multiply them by GDP. The expenditure is reported quarterly. Therefore I assume that the annualized shares are the same. This information provides the consumption expenditures including consumption taxes. Since it is assumed that X is not taxed, then for this good the consumption expenditure is equal to the output value. From the IES I also obtain the income share from land and resources. Table 4.2 shows the levels and shares of consumption for each good and income shares by income decile. I define a new parameter, ψ_{in}^{h}, as the income of group h as share of total income. This share parameter shows the distribution of income across income groups.

The income shares on food are what one would expect; low-income groups spend a higher share of their income on necessities such as food. Expenditure shares on food decrease almost monotonically from the first to the tenth decile. The expenditure shares on clean manufacturing increase monotonically with income. This is not surprising, as most clean manufacturing goods are not considered a necessity. High-income groups have a higher income share from land rents. The expenditures share on energy do not change significantly across income groups. This contrasts with other countries, where it has been found that the expenditures shares on energy goods is higher for the low-income groups. A possible explanation is that low-income groups in developing countries do not own cars or houses with available utilities where energy goods are directly consumed. In our case, from the data shown in Table 4.2, I can see that high-income groups have higher expenditures shares in clean manufacturing, lower expenditure shares in agricultural goods and higher income shares from land rents.

Table 4: Expenditure on Food, Clean and Dirty Manufacturing and Income Sources by Income Decile

	MILLION OF PESOS			EXPENDITURE SHARES			INCOME SHARES		
Income Decile	Food	Dirty Manuf.	Clean Manuf.	Food	Dirty Manuf.	Clean Manuf.	Total Income	Land	Resources
	$P_X X^h$	$(P_Y + t)Y^h$	$(P_Z + t)Z^h$	ψ_X^h	ψ_Y^h	ψ_Z^h	ψ_{in}^h	ψ_A^h	ψ_R^h
I	52, 010	9, 438	42, 523	0.5002	0.0908	0.4090	0.0190	0.0045	0.9955
II	88, 566	16, 070	72, 116	0.5011	0.0909	0.4080	0.0322	0.0046	0.9954
III	111, 604	22, 672	105, 123	0.4662	0.0947	0.4391	0.0436	0.0049	0.9951
IV	123, 334	26, 385	127, 884	0.4443	0.0950	0.4607	0.0506	0.0069	0.9931
V	145, 689	34, 337	169, 463	0.4169	0.0982	0.4849	0.0637	0.0064	0.9936
VI	153, 670	44, 886	201, 640	0.3840	0.1122	0.5039	0.0730	0.0070	0.9930
VII	179, 484	54, 527	269, 379	0.3566	0.1083	0.5351	0.0918	0.0081	0.9919
VIII	209, 357	68, 032	351, 874	0.3327	0.1081	0.5592	0.1147	0.0159	0.9841
IX	241, 335	94, 014	496, 113	0.2903	0.1131	0.5967	0.1516	0.0124	0.9876
X	336, 017	172, 063	1, 465, 769	0.1702	0.0872	0.7426	0.3598	0.0339	0.9661
Total	1, 641, 065	542, 424	3, 301, 883	0.2992	0.09889	0.6019	1	0.0183	0.9816

Data on the amount of government spending are obtained from the Informe Estadistico del Presidente 2002. Government spending was 15% of GDP in 2000. Hence by multiplying this figure by the GDP I get the value of government spending.

In order to obtain the output value (P_iQ_i) for Y and Z and the tax rate I need to perform a simple iteration. From the IES and from the government budget constraint I know the value of $(1+t)Z$, $(1+t)Y$, and $tZ+tY$. I have two independent equations and three unknowns: Y, Z, t. I start with a guess of $t = 0.15$, which will be the value if all goods were taxed at the same rate, and obtain the estimated Y and Z. Then I substitute the estimated tax and quantities into $tZ+tY$ and check if the estimated sum is equal to the known value. If not, I increase the tax until convergence is achieved. The result is shown in Table 5

I use the zero profit conditions for X and Z to obtain the value of resources used in both industries, R_X and R_Z. Note that from the IES I know the amount of land used in the production of X. From Salhofer (2000) I obtain the mean elasticity of substitution between labor and land and capital and land for 32 studies. The elasticity of substitution in X is set equal to the average of both elasticities.

Finally, as in Fullerton and Metcalf (2001), I start by assuming that the dirty good produces externalities either in consumption or production. I believe that given our definition of Y, this is a good approximation. Hence, the production of Y is simplified to $Y = R_W$. That implies $\sigma_Y = 0$ and $\phi = 0$. The parameter values are shown in Table 5.

Table 5: Parameter Values

Expenditure Shares	Input Shares	Taxes
$\psi_X = 0.29917$	$\phi = 0$	$t = 0.29107$
$\psi_Y = 0.09889$	$\theta = 0.95865$	$\gamma = 0.77455$
$\psi_Z = 0.60194$		$\delta = 0.22544$

4.2.1 Estimation of Utility Function Parameters

The parameters of the utility function were estimated using data from IES for Mexico. The IES consists of interviews to 11,170 households. Ideally, I would want to have price and quantity data. However, the IES provides price data only for a small number of purchases. Therefore, I assume that all individuals face the same prices and normalize all prices to one. Clearly, the estimated parameters may be biased if individuals face different prices.

I aggregate the expenditure data to match our definitions of X, Y and Z for the 11,170 households. Households that do not report expenditures for any of the three goods were not considered. Hence, I end up with 9,699 households in our sample. Income is calculated as the sum of expenditures for all goods. I also adjust for household size. A common approach is to divide household expenditures by the number of household members over 18 years and expand the number of observations for each household by this number.[10] This approach ignores the possible economies of scale in household consumption.[11] I adjust by household size by including the number of household members over 18 years as an explanatory variable. Note that the equations are likely to be related making it a system of seemingly unrelated equations (SUR). However, since all equations have the same explanatory variables I can estimate the parameters by performing OLS on each equation.[12] In order to obtain the aggregate "minimum consumption" parameter I multiply the estimated constant term times the total number of households (9,699) and add that to the estimated coefficient on household size times the number of household members (23,499). Next, I calculate the respective elasticities. The results are summarized in Table 6.

Since I do not have data on the income from land for each household, the elasticity with respect to the price of land is obtained by multiplying the income elasticity times the overall share of land income on total income. The values in the main diagonal of the first three columns in the bottom section of Table 6 represent the uncompensated own-price elasticity. The off-diagonal elements are the uncompensated cross-price elas-

[10] I also obtain the elasticities using this approach. The elasticities are similar to the ones presented in Table 6

[11] See Deaton and Muellbauer (1980) and Pollak and Wales (1992).

[12] See Greene, H. William (1997).

Table 6: LES Estimated Parameters and Resulting Elasticities

	Estimated		Implied Elasticities				
	α_i	$\bar{Q}_i^a$	$\varepsilon_{i,X}$	$\varepsilon_{i,Y}$	$\varepsilon_{i,Z}$	$\varepsilon_{i,in}$	ε_{i,P_A}
X	0.11230	28,008,759	-0.4282	-0.0349	0.0814	0.3558	0.0044
Y	0.07263	3,501,083	-0.1506	-0.7598	0.1695	0.7409	0.0092
Z	0.81507	-31,509,840	-0.4387	-0.0353	-1.0727	1.3900	0.0172

ticities. The low own-price elasticity of X is consistent with the fact that this good is a necessity. King (1979) found an own-price elasticities of food of -0.53. The own-price elasticity of our energy good is in the upper limit of the range estimated for gasoline. Also it is higher than the elasticity for utilities in King (1979).[13] The relative high own-price elasticity of Z is expected, as it is not considered a necessity.

As pointed out in Section 4.1, price and income elasticities are approximately proportional. Note that the ordering of income elasticities is the same as the ordering of own-price elasticities.[14] The estimated income elasticity of food is lower than the one estimated by Decoster and Schokaert (1990) and King (1979) of 0.53 and 0.50 respectively. The income elasticity of energy is within the range of 0.33 and 0.86 with a mean of 0.64 for gasoline presented in Espey (1996).

The cross-price elasticities with respect to P_X and P_Y are negative, which implies that the income effect dominates the substitution effect in these cases.[15] Finally, the cross-price elasticities with respect to P_Z are positive, which indicates that the substitution effect dominates the income effect, making Z a relatively good substitute for X and Y.

[13] These elasticities were updated by Fullerton and Rogers (1993).

[14] This is one of the restriction implicitly imposed on the LES, see Deaton and Muehbauer (1980) for an overview.

[15] If I observe the compensated elasticities in Appendix D, I can see that X and Y are not good substitutes for any other good.

5 Numerical Results

I examine the general equilibrium effects of introducing emission taxes. I present the numerical results for the prices, taxes and emissions, since these are the only values needed to obtain the distributional effects (see Eq. (2.26)).

Table 7: Numerical Results in Prices and Taxes

Prices and Taxes	Quantities
$\widehat{t} = -0.11120\widehat{t}_w$	$\widehat{Z} = 0.096\widehat{t}_w$
$\widehat{P}_A = -0.09227\widehat{t}_w$	$\widehat{X} = -0.031\widehat{t}_w$
$\widehat{P}_X = -0.00381\widehat{t}_w$	$\widehat{Y} = -0.52\widehat{t}_w$
$\widehat{P}_Y = \widehat{t}_W$	$\widehat{W} = -0.52\widehat{t}_W$

Given that the initial tax on emissions is zero, then $\hat{t}_W = dt_W$. Table 7 tells us the percentage change in prices, manufacturing taxes and quantities from a change in the environmental tax. As expected, an increase in the environmental tax reduces the pre-existing tax on manufacturing. The producer price of Y increases by the same amount. Using the data from the Table 7 and the original price and tax, I know that the consumer price of Y increases.[16] The consumer price of the clean manufacturing decreases as the manufacturing tax decreases. The cross-price elasticity between X and Y is small but negative. Hence, an increase in the price of dirty manufacturing reduces the demand for agricultural products. Also, the cross-price elasticity between X and Z is small but positive. A decrease in the price of clean manufacturing decreases the demand for agricultural goods. The reduction in pollution produces a fall in the amount of resources used in W. The increase in the quantity demanded of the clean good yields a higher demand for resources in that sector. The amount of resources released by W is not enough to compensate for the increase in the demand for inputs in sector Z. Hence, the amount of resources used in agriculture falls. This produces a drop in the production of agriculture (see Eq. (2.18)). A lower use of resources and production in agriculture reduces the price of land (see Eq. (2.20)). Since the price of

[16] The change in the consumer price of Y is given by the following expression: Δ Consumer Price of $Y = \gamma\widehat{P}_Y + \widehat{t} = 0.66335\widehat{t}_W$

resources is fixed, the price of agriculture falls as a result of the drop in the price of land.

The log-linearization is useful for marginal changes. Thus, I assume a tax increase of 1% (a tax of 0.01). The consumer price of Y goes up by 0.663%. The consumer price of Z goes down by the change in the tax, 0.111%. The price of X goes down by 0.004%. Emissions are reduced by 0.52%. Following Fullerton and Metcalf (2001) I set the overall marginal environmental damages (ϱ) equal to 0.1. The overall change in welfare by its components are shown in Table 8.

Table 8: Total Welfare Change by its Components *

	Land Rents	Manuf. Tax	Food Price	Dirty Manuf.	Real Income	Benefits	Welfare
Total	- 0.169	7.794	0.114	- 7.659	0.080	1.202	1.282

$*$ Values are multiplied by 10, 000 to make them easier to read.

The overall welfare increases with the policy.[17] A one percent increase in the environmental tax yields an increase in welfare that is equal to 0.013% of national income. The overall real income increases by 0.0008%. Note that most of the increase in the overall real income comes from the drop in the manufacturing tax.

The change in the real income of each income group can be observed in Table 9. As mentioned earlier, I do not consider the distribution of the benefits across income groups.

The deciles are ordered from the first being to lowest income group to the tenth being the richest group. The costs of the policy are regressive. From Table 9, I can observe that the low-income groups receive a higher gain from the reduction in the price of food. The third column shows the effect of lower manufacturing taxes. High-income groups receive a higher benefit from lower manufacturing taxes. The fifth column shows the effect of a higher producer price for Y. The income groups more affected by this are the high-income groups, with the exception of the tenth decile. The most important effects on the magnitude of the costs are the reduction in manufacturing taxes and the increase in the producer price of Y. The change in real income of each

[17] According to the definition of *ameb*, this means that $ameb < 0$.

Table 9: Change on Real Income by its Components by Income Decile *

Income Decile	Land Rents	Manuf. Tax	Food Price	Dirty Manuf.	Real Income
I	-0.041	5.558	0.191	-7.031	-1.323
II	-0.043	5.549	0.191	-7.042	-1.345
III	-0.045	5.937	0.178	-7.335	-1.266
IV	-0.064	6.181	0.170	-7.362	- 1.075
V	-0.059	6.485	0.159	-7.610	- 1.025
VI	-0.064	6.851	0.146	-8.687	- 1.754
VII	-0.074	7.156	0.136	-8.390	-1.172
VIII	-0.146	7.421	0.127	-8.374	- 0.972
IX	-0.115	7.894	0.111	-8.758	-0.868
X	-0.313	9.228	0.065	-6.752	2.229

* Values are multiplied by 10, 000 to make them easier to read.

group is given by the sum of the second to fifth column. Real income decreases for the first nine income deciles while it increases for the tenth decile. The sum of the change in real income for each income decile weighted by its respective income share yields the aggregate change in real income shown in the seventh column of Table 8.

The importance of distributional effects is clear in this case. All income groups with exception of the tenth decile bear a positive cost. That is, the real income of these groups decreases. However, the overall cost of the policy is negative, - i.e. overall real income increases. This is a result of the high weight that the tenth decile has on the aggregate income.

The distribution of the costs of the policy are explained by several factors. Income groups with high expenditures in the clean manufacturing will tend to gain more from the policy as the lower manufacturing taxes effect dominate. This is true for the richest 30% of the households (tenth, ninth and eighth decile). The income groups that bear the highest costs have either high consumption shares in the dirty good and low expenditure shares in the clean manufacturing or very low shares in clean manufacturing. This holds for the bottom four deciles. Note that the sixth and ninth deciles have highest expenditure shares in Y, however the ninth decile has also high expenditure shares in Z. As a result the ninth decile receives the second highest benefit and the sixth decile bears the highest cost. The income groups that receive a medium cost are the groups with medium expenditure shares in Y and Z. These are the fifth, fourth and seventh deciles. The income effects from land are not as important as expected because in the data, the income groups with the highest land shares, where these effects will tend to be more important, are also the groups with the highest expenditure shares for clean manufacturing. Therefore, the reduction in the consumer price of the clean manufacturing good dominates the income effect from land. Moreover, the income shares from land rents are low. In economies where agriculture is more predominant than in Mexico, I can expect the income effect to be more important.

The environmental tax by itself is not regressive. The regressivity is driven by the cut in the manufacturing tax rather than the pollution tax. The expenditure share in the manufacturing sector increases almost monotonically with income. Hence, high-income groups receive higher benefits from the tax cut in manufacturing. If the revenue obtained from the environmental tax is returned in a different way, the distributional effect could be different. For instance, the government could return the tax as a subsidy to agricultural products or as a lump-sum subsidy.

The policy will be progressive only if the benefits are progressive. In the case where benefits are progressive the welfare gain for each income group would depend on the magnitude of cost and benefits (with exception of the tenth decile).

The results in Tables 8 and 9 depend on the specific parameter values used in this model. In particular I want to know the sensitivity of the results to $\phi > 0$. That is, a lower share of pollution in the production of dirty manufacturing. The following tables show the results for values of ϕ equal to 0.1 and 0.2, and $\sigma_Y = -1$.

A lower share of pollution in the production of Y is translated into a lower effect in its price and a higher effect in Z. Real income, benefits and welfare are lower in both cases with respect to the original case. That is, the impact of the policy is lower as the pollution level decreases.

The distributional results on the real income are shown in Table 11. Since the price of Y does not increase as much as before, groups with high expenditure shares in Y

Table 10: Sensitivity Analysis of Total Welfare, Benefits and Real Income*

	$\sigma_Y = -1, \phi = 0.1$	$\sigma_Y = -1, \phi = 0.2$
Real Income	0.072	0.064
Benefits	1.180	1.137
Welfare	1.252	1.201

∗ Values are multiplied by 10, 000 to make them easier to read.

will be hurt less when ϕ is larger.

Table 11: Sensitivity Analysis of Real Income Results by Income Decile*

Income Decile	$\sigma_y = -1, \phi = 0.1$ Real Income	$\sigma_y = -1, \phi = 0.2$ Real Income
I	-1.191	-1.0587
II	- 1.210	- 1.0757
III	- 1.139	- 1.0125
IV	- 0.968	- 0.860
V	- 0.922	- 0.819
VI	- 1.579	- 1.403
VII	- 1.055	- 0.937
VIII	- 0.875	- 0.777
IX	- 0.781	- 0.695
X	2.006	1.783

∗ Values are multiplied by 10, 000 to make them easier to read.

The distributional effects are robust to the different values of ϕ. The distribution of the costs are similar to the previous case. High-income groups bear the lowest costs. Whereas the low-income groups together with the sixth decile bear the highest costs of the policy. The distributional effects change slightly in magnitude but not in order.

Finally, it is possible that the distributional effects can change for different elasticities. However, performing sensitivity analysis implies changing the estimated parameter values and that is out of the scope of this work.

6 Conclusions

Most empirical studies have shown that environmental taxes are regressive and that some kind of compensation must be used to reduce the regressivity of the tax. The present study, calibrated with Mexican data, finds that the distribution of the costs from the combination of an environmental tax and a cut in manufacturing taxes is mostly regressive. The regressivity of the tax is driven not by the environmental tax but rather by the tax cut on manufacturing. If the extra revenue from the environmental tax is returned in a different way, the distributional effects of the policy could be different.

In contrast to other studies, the income groups that pay a higher cost are not necessarily the groups with higher expenditure shares in the dirty good. High-income groups will bear lower costs of the policy since they have higher expenditure shares in clean manufacturing. Moreover, income groups with either high expenditure shares in the dirty manufacturing good and low expenditure shares in the clean manufacturing goods or very low expenditure shares in clean manufacturing bear higher costs. These are low-income groups and the sixth decile. The richest 30% of the households bear the lowest burden of the tax. Moreover, households in the top 10% bear a positive burden as their real income increases after the policy. The sixth decile and the lowest 30% of the households bear the highest burden. The fifth, fourth and seventh income decile bear a medium burden.

In this study, I observe a trade off between efficiency and equity. The aggregate welfare increases but at the costs of a less equitable distribution of income. However, without reliable estimates on the distribution of benefits, it is impossible to obtain the distribution of welfare across income groups. This is not a trivial issue, if benefits are progressive then the overall effect of the policy on welfare may be progressive. However, the regressivity of the policy may be exacerbated if benefits are regressive.

The income effects from land are relatively small given our parameter values for Mexico. Income effects are more important for countries where land rents represent a higher share of income. The price of land decreases and that reduces the regressivity in the distribution of the costs.

Finally, the distribution of the costs appears to be robust to different values of the pollution to production of the dirty good ratio.

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Appendix A

Deaton and Muellbauer (1982), show that the condition for exact aggregation can be stated in terms of the expenditure function. The general condition is the following.

$$e(U,P) = \theta(U, a(p), b(p)) \tag{A-1}$$

Where a(p), b(p) and θ are linear homogeneous functions of prices and θ is linearly homogeneous in a and b [18].

After some manipulation we obtain the condition in the expenditure share:

[18] For example a(p)= $\sum a_k P_k$ and b(p) =$\beta_o \prod {P_k}^{\beta_k}$

$$\overline{W_i} = (1-\lambda)\frac{\partial log(a)}{\partial log(P_i)} + \lambda\frac{\partial log(b)}{\partial log(P_i)} \tag{A-2}$$
$$\text{where} \qquad \lambda \equiv \frac{\partial log(\theta)}{\partial log(b)}$$

Note that the linear expenditure system satisfies these conditions.

Appendix B

Linearization of the System

In this Appendix I will show how the original model was linearized to obtain Equations (2.15) - (2.25).

In order to obtain Equations (2.15) - (2.17) total differentiate Eq. 2.3 divide both sides by Q_i and multiply each term on the right hand side by the relevant price.

$$\frac{dQ_i}{Q_i} = \frac{\partial Q_i}{\partial P_X}\frac{P_X}{Q_i}\frac{dP_x}{P_X} + \frac{\partial Q_i}{\partial(P_Y+t)}\frac{(P_Y+t)}{Q_i}\frac{P_Y}{(P_Y+t)}\frac{dP_y}{P_Y} + \frac{\partial Q_i}{\partial(P_Z+t)}\frac{(P_Z+t)}{Q_i}\frac{P_Z}{(P_Z+t)}\frac{dP_z}{P_Z}$$
$$+ \frac{\partial Q_i}{\partial(P_Y+t)}\frac{(P_Y+t)}{Q_i}\frac{dt}{(P_Y+t)} + \frac{\partial Q_i}{\partial(P_Z+t)}\frac{(P_Z+t)}{Q_i}\frac{dt}{(P_Z+t)} + \frac{\partial Q_i}{\partial P_A}\frac{P_A}{Q_i}\frac{dP_a}{P_A}$$
$$\frac{\partial Q_i}{\partial A}\frac{A}{Q_i}\frac{dA}{A}$$

setting $P_X = P_Y = Pz = 1$, $dA = dP_z = 0$, $\varepsilon_{iX} = \frac{\partial Q_i}{\partial P_X}\frac{P_X}{Q_i}$,$\varepsilon_{iY} = \frac{\partial Q_i}{\partial(P_Y+t)}\frac{P_Y}{Q_i}$, $\varepsilon_{iZ} = \frac{\partial Q_i}{\partial(P_Z+t)}\frac{(P_Z+t)}{Q_i}$, and $\varepsilon_{iA} = \frac{\partial Q_i}{\partial P_A}\frac{P_A}{Q_i}$, I can re-write the previous equation as follows

$$\widehat{Q}_i = \varepsilon_{iX}\widehat{P}_X + \varepsilon_{iY}\frac{1}{(1+t)}\widehat{P}_Y + \varepsilon_{iY}\widehat{t} + \varepsilon_{iZ}\widehat{t} + \varepsilon_{iA}\widehat{P}_A$$

Now, set $\gamma = \frac{1}{(1+t)}$ rearrange and I obtain Equations (2.15) to (2.17)

$$\widehat{Q}_i = \varepsilon_{iX}\widehat{P}_X + \varepsilon_{iY}(\gamma\widehat{P}_Y + \widehat{t}) + \varepsilon_{iZ}\widehat{t} + \varepsilon_{iA}\widehat{P}_A \qquad \forall \qquad i = X, Y, Z \tag{B-1}$$

In order to obtain Eq. (2.18) total differentiate Eq. (2.4) divide by X on both sides and multiply and divide each term on the right hand side by the variable being differentiated.

$$\frac{dX}{X} = \frac{dR_X}{R_X}\frac{R_X}{X} + \frac{dA}{A}\frac{A}{Y}$$

Set $dA = 0$ and $\theta = \frac{R_X}{X}$

$$\widehat{X} = \widehat{R}_X \theta \tag{B-2}$$

I use the same procedure to Eq. (2.5) to obtain Eq. (2.19).

$$\frac{dY}{Y} = \frac{dR_Y}{R_Y}\frac{R_Y}{Y} + \frac{dW}{W}\frac{W}{W}$$

Set $\phi = \frac{R_Y}{Y}$

$$\widehat{Y} = \widehat{R}_W \phi + \widehat{W}(1-\phi) \tag{B-3}$$

In order to obtain information about the substitution of R_X and A in the production of X, I used the log-linearization method in the elasticity of substitution equation. As a reminder the elasticity of substitution is defined as:

$$\sigma_X = \frac{d[\frac{A}{R_X}]/\frac{A}{R_X}}{d[\frac{P_A}{P_R}]/\frac{P_A}{P_R}}$$

Taking the derivative of the term in the numerator, noting that $P_R = 1$ (numeraire) and rearranging a little bit I obtain Eq. (2.20).

$$\begin{aligned} \sigma_X &= \frac{\{\frac{dA}{R_X} - \frac{AdR_x}{R_X{}^2}\}\frac{R_X}{A}}{\widehat{P}_A} \\ \sigma_X &= \frac{\widehat{A} - \widehat{R}_X}{\widehat{P}_A} \\ \widehat{R}_X &= -\sigma_X \widehat{P}_A \qquad \text{where} \qquad \sigma_X < 0 \end{aligned} \tag{B-4}$$

Applying the same procedure for σ_y and taking in consideration that $P_W = (1 + t_W)$ and $\widehat{P}_w = 0$, I obtain Eq. 2.21.

$$\begin{aligned} \sigma_y &= \frac{\widehat{R}_Y - \widehat{W}}{\widehat{P}_R - \widehat{W}} \\ \widehat{R}_y - \widehat{W} &= -\sigma_y \widehat{t}_w \qquad \text{where} \qquad \sigma_y < 0 \end{aligned} \tag{B-5}$$

Eq. (2.22) was derived from the resource constraint given by Eq. (2.10).

$$\frac{dR}{R} = \frac{dR_X}{R_X}\frac{R_X}{R} + \frac{dR_Y}{R_Y}\frac{R_Y}{R} + \frac{dR_Z}{R_Z}\frac{R_Z}{R} + \frac{dR_W}{R_W}\frac{R_W}{R} + \frac{dR_G}{R_G}\frac{R_G}{R}$$

Since the total level of resources is fixed then $dR = 0$. Moreover, given that government spending does not change that implies $R_G = 0$. Setting $\mu = \frac{R_j}{R}\ \forall\ j = X, Y, Z, W, G$ I obtain Eq. (2.22).

$$0 = \widehat{R}_X \mu_X + \widehat{R}_Y \mu_Y + \widehat{R}_Z \mu_Z + \widehat{R}_W \mu_W \tag{B-6}$$

Eq. (2.23) is obtained by total differentiating the government budget constraint and setting $dG = 0$ because the government keeps a balanced budget and $t_W = 0$ since the initial tax on pollution is zero.

$$0 = Yt\frac{dY}{Y} + Y(1+t)\frac{dt}{(1+t)} + Zt\frac{dZ}{Z} + Z(1+t)\frac{t}{1+t} + W(1+t_W)\frac{dt_W}{(1+t_W)}$$

Dividing both sides by income

$$0 = \frac{Yt}{P_AA + P_RR}\widehat{Y} + \frac{Y(1+t)}{P_AA + P_RR}\widehat{t} + \frac{Zt}{P_AA + P_RR}\widehat{Z} + \frac{Z(1+t_z)}{P_AA + P_RR}\widehat{t} + \frac{W(1+t_W)}{P_AA + P_RR}\widehat{t_w}$$

Setting $\delta = \frac{t}{1+t}$ and $\psi_i = \frac{Q_i(1+t)}{P_AA+P_RR}$ I obtain Eq. (2.23)

$$0 = \delta\psi_Y\widehat{Y} + \psi_Y\widehat{t} + \delta\psi_Y\widehat{Z} + \psi_Z\widehat{t} + \psi_W\widehat{t_W} \tag{B-7}$$

In a competitive equilibrium firms profits will be zero. Using the log-linearization method on the Eq. (2.12) I obtain Eq. (2.24)

$$\begin{aligned} X\widehat{P}_X + X\widehat{X} &= P_RR_X\widehat{R}_X + AP_A \\ \widehat{P}_X + \widehat{X} &= \frac{P_RR_X}{X} + \frac{AP_A}{X}\widehat{P}_a \\ \widehat{P}_X + \widehat{X} &= \theta\widehat{R}_X + (1-\theta)\widehat{P}_A \end{aligned} \tag{B-8}$$

Similarly, Eq. (2.25) comes from linearizing the zero-profit condition for Y given by Eq. (2.13).

$$\widehat{Y} + \widehat{P}_Y = \frac{R_YP_R}{Y}\widehat{R}_Y + \frac{W(1+t_W)}{Y}\widehat{t}_W + \frac{W(1+t_W)}{Y}\widehat{W}$$

Substitution the initial tax on pollution, $t_W = 0$, and the previous definition of ϕ

$$\widehat{Y} + \widehat{P}_Y = \phi\widehat{R}_Y + (1-\phi)(\widehat{W} + \widehat{t}_W) \tag{B-9}$$

Appendix C

Marginal Excess Burden

I derive the *meb* following De Mooij (2001). I define the *meb* as the compensating variation divided by income. In order to obtain the compensating variation I total differentiate the utility function of each income group.

$$dU^h = \frac{\partial U^h}{\partial X^h}dX^h + \frac{\partial U^h}{\partial Y^h}dY^h + \frac{\partial U^h}{\partial Z^h}dZ^h + \frac{\partial U^h}{\partial E}\frac{\partial E}{\partial W}dW + \frac{\partial U^h}{\partial G}dG \tag{C-1}$$

The compensating variation is defined as the amount of money at the new prices necessary to bring the consumer back to its original level of utility. Hence, utility does not change $dU^h = 0$. In addition, government spending does not change, that is $dG = 0$. From the consumer problem I obtain the first order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial X^h} &= \frac{\partial U^h}{\partial X^h} - \lambda P_X = 0 && \text{(C-2)}\\
\frac{\partial \mathcal{L}}{\partial Y^h} &= \frac{\partial U^h}{\partial Y^h} - \lambda (P_Y + t) = 0 && \text{(C-3)}\\
\frac{\partial \mathcal{L}}{\partial Z^h} &= \frac{\partial U^h}{\partial Z^h} - \lambda (P_Z + t) = 0 && \text{(C-4)}\\
\frac{\partial \mathcal{L}}{\partial \lambda^h} &= A^h P_A + R^h P_R - X^h P_X - (P_Y + t) Y^h - (P_Z + t) Z^h = 0 && \text{(C-5)}
\end{aligned}$$

Substituting first order conditions (C-2) to (C-4) into Eq. (C-1) I obtain the following expression

$$0 = \lambda P_X dX^h + \lambda (P_Y + t) dY^h + \lambda (P_Z + t) dZ^h + \frac{\partial U^h}{\partial E} \frac{\partial E}{\partial W} dW \tag{C-6}$$

Next, I total differentiate the household budget constraint, add the variable CV^h for the compensating variation for each income group, and set $dA = dP_Z = dP_R = dR^h = 0$.

$$\begin{aligned}
A^h dP_A + CV^h = X^h dP_X + P_X dX^h + Y^h dP_Y + Y^h dt \\
+ (P_Y + t) dY^h + Z^h dt + (P_Z + t_Z) dZ^h
\end{aligned} \tag{C-7}$$

Substitute Eq. (C-7) into Eq. (C-6) and solve for CV^h.

$$CV^h = -A^h dP_A + Z^h dt + Y^h (dP_Y + dt) + X^h dP_X - \frac{\partial U^h}{\partial E} \frac{\partial E}{\partial W} \frac{1}{\lambda^h} dW \tag{C-8}$$

Divide both sides by income and define $\varrho^h = -\frac{\partial U^h}{\partial E} \frac{\partial E}{\partial W} \frac{1}{\lambda^h}$.

$$\begin{aligned}
\frac{CV^h}{A^h P_A + P_R R^h} &= \frac{A^h P_A}{A^h P_A + P_R R^h} \frac{dP_A}{P_A} + \frac{(1+t) Z^h}{A^h P_A + P_R R^h} \frac{dt}{(1+t)} \\
&+ \frac{P_Y Y^h}{A^h P_A + P_R R^h} \frac{dP_Y}{P_Y} + \frac{(1+t) Y^h}{A^h P_A + P_R R^h} \frac{dt}{(1+t)} \\
&+ \frac{X^h P_X}{A^h P_A + P_R R^h} \frac{dP_X}{P_X} + \frac{W \varrho^h}{A^h P_A + P_R R^h} \frac{dW}{W}
\end{aligned}$$

Simplifying and using the shares defined previously I obtain Eq. (2.26).

$$meb^h = -\psi_A^h \widehat{P}_A + \psi_Z^h \widehat{t} + \psi_Y^h \gamma \widehat{P}_Y + \psi_Y^h \widehat{t} + \psi_X^h \widehat{P}_X + \psi_W^h \varrho^h \widehat{W}$$

Rearranging

$$meb^h = -\psi_A^h \widehat{P}_A + \widehat{t}(1 - \psi_X^h) + \psi_X^h \widehat{P}_X + \gamma \psi_X^h \widehat{P}_Y + \psi_W^h \varrho^h \widehat{W} \tag{C-9}$$

Note that by following the same procedure for the aggregate utility I obtain the *ameb* in Eq. (2.27).

Appendix D

In this Appendix we show the compensated elasticities obtained.

Table D-1: LES Compensated Elasticities

	$\varepsilon_{i,x}$	$\varepsilon_{i,y}$	$\varepsilon_{i,z}$
X	-0.31585	0.07743	0.29001
Y	0.08325	-0.68719	0.60394
Z	0.15617	0.10093	-0.25710