The Confusing Time-Series Behaviour of Real Exchange Rates: Are Asymmetries Important?

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Abstract

Evidence regarding the time-series properties of real exchange rates is mixed. There is evidence that such rates exhibit both non-stationary and stationary behaviour. The current dominant belief is that rates are non-linear stationary, however, this is not accepted without question. This paper re-examines the time-series properties of five US Dollar real exchange rates and argues that the confusing time-series properties arise largely as each series examined exhibits periods of non-stationary and stationary behaviour such that the sample over which any empirical exercise is conducted is of importance. However, extending a typical non-linear model used within the literature to allow for asymmetries improves the models ability to fit the data. As such our results suggest that modelling asymmetries between positive and negative real exchange rate deviations is of importance, whereas extant research has typically rules out asymmetry. Indeed a forecasting exercise conducted over a one-year horizon is particularly supportive of this model. Such a finding is of importance not only for academics but also finance practitioners involved in trading and portfolio management and finance managers who act in the foreign exchange market for goods market trading. It remains for future research to theoretically motivate the asymmetries found here.

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1. Introduction.

The behaviour of real exchange rates remains a hot topic within the empirical finance literature, largely because of a significant number of papers supporting and rejecting real exchange rate stationarity. Recent research has focussed upon the belief that real exchange rates are stationarity but exhibit non-linear dynamics in the process of mean reversion. In particular, the belief is that for small deviations from mean (equilibrium) value real exchange rates are non-stationary due to the presence of market frictions, transaction costs and other costs of international goods market and financial arbitrage, while larger deviations do exhibit mean reversion. Two popular such models are the band TAR model of Obstfeld and Taylor (1997) and the restricted ESTAR model of Taylor, Peel and Sarno (2001) both of which model the inner and outer regimes according to different autoregressive processes. Moreover, however, there are a variety of other models all claiming to accurately describe the dynamics of international real exchange rates, for which examples include the modified LSTAR model (Sollis et al, 2002), the non-linear trend-stationary model (Sollis, 2005), the multiple-regime STAR model (Bec et al, 2004) and the fractionally-integrated STAR model (Smallwood, 2005). Whilst, more recent further evidence supporting non-linear reversion is proved by Paya et al (2003), Chortareas and Kapetanios (2004) and Paya and Peel (2007).

However, despite this evidence there is not yet a consensus regarding the behaviour of real exchange rates. On the one hand several papers maintain that real exchange rates are non-stationary (for example, Engel, 2000; Cuddington and Liang, 2000; Arghyrou and Gregoriou, 2007). On the other hand, whilst the above non-linear models may provide a reasonable in-sample fit for a selection of real exchange rates they have limited forecasting power (see, in particular, Rapach and Wohar, 2006).[i]

The present paper, therefore, re-examines the behaviour of five major real exchange rate series, all against the US dollar, and aims to show that the confusing empirical behaviour of real exchange rate series largely arises because most real exchange rate series exhibit both random walk and stationary behaviour that changes over time, such that different sample periods will produce different empirical results. Nonetheless, the results from this paper suggest that accounting for asymmetries within real exchange rates may be important for understanding their movements and to obtain improved forecasts. This latter point being particularly important for both market traders and finance managers whereby improved forecasting power can facilitate trading and hedging strategies.

More specifically, we estimate an asymmetric version of the popular ESTR model and show that each of the real exchange rate series exhibit some form of asymmetry as well as non-linearity in the process of mean reversion. Furthermore, this model typically provides for improved forecasts over alternate linear and non-linear models at the one-year horizon, although as a note of caution it is arguable that this model is not able to explain the dynamics of all five series (notably for Canada), such that there is no 'one-size-fits-all' model as a different model maybe required to capture the individual nuances of different rates. The remainder of the paper is organised as follows: Section 2 presents a brief background to the real exchange rate debate and some current literature; Section 3 introduces the data and conducts the linear and non-linear tests previously examined within the literature; Section 4 considers a revised version of the ESTR model that allows for asymmetries with real exchange rate dynamics; Section 5 presents a discussion of our results and considers a forecasting exercise; Section 6 summarises and concludes.

2. Background and Brief Review.

Theoretical discussion real exchange rate dynamics centres around two possible behaviours. First, the theory of PPP states that the exchange rate between two countries should be such that the two national price levels if expressed in a common currency at the prevailing rate should equate. Hence, the purchasing power of a unit of either currency is identical. As such any deviations from PPP should be short-lived and exhibit reversion. Defining the real exchange rate as the measure of deviation from PPP then its time-series properties should be characterised by stationarity. More explicitly, if PPP held continuously then the real exchange rate would be a constant, however, whilst we would not expect that to hold in the real world there is a common belief that PPP nevertheless acts as an anchor for real exchange rate movements. Support for such a belief has been provide by Froot and Rogoff (1995), Taylor (1995), Rogoff (1996) and Sarno and Taylor (2002).

In contrast, a second literature that dates back to Roll (1979) and Adler and Lehmann (1983), argues that if international financial markets are efficient then deviations from PPP should not exhibit any tendency to revert to a long-run attractor point but instead follow a random walk. Evidence of such non-stationary behaviour has been provided by, amongst others, Meese and Rogoff (1988), Enders (1988), Taylor (1988) and Mark (1990). More recently, Taylor and Sarno (2004) have attempted to reconcile these two theoretical strands by relaxing the assumption of a constant real interest rate differential made in the Roll and Adler and Lehmann models.

Much of the recent attention regarding the time-series properties of real exchange rates has focussed upon possible non-linear dynamics as the source of apparent non-stationarity. In particular, if the process is truly non-linear than applying linear unit root tests, which by definition ignore this property, will severely affect the power of such tests resulting in an inability to reject the null. Evidence in favour of non-linear stationary behaviour has been reported by, amongst others, Michael et al (1997), Obstfeld and Taylor (1997), Taylor (2001) and Taylor et al (2001). Given the belief that real exchange rates may follow non-linear dynamic behaviour it is important to provide some rationale for such behaviour. The main reason advanced is that the non-linearity arises from the presence of transaction costs that affect the efficiency of goods market arbitrage. Indeed formal models of international goods market arbitrage (Dumas, 1992, 1994; Sercu et al, 1995) support this view whereby costly arbitrage results in both delays in price adjustment and non-linear effects. Alternative explanations for non-linear mean reversion arise from policy intervention or the actions of market agents in financial speculation and arbitrage behaviour. This view was expressed by Taylor and Allen (1992), where the market and policy makers may view small deviations from equilibrium as relatively unimportant, and only once deviations become large will pressure from the market and policy makers be exerted in order to restore fundamental equilibrium. Furthermore, they argue that non-linearities may explain the use of technical analysis amongst foreign exchange market traders, which necessitates close examination of market movements. More recently, Dutta and Leon (2002) have also suggested that non-linearities may arise due to policy intervention. That is, policy authorities may curb excessive depreciations and appreciations due to their possible negative affects upon the macroeconomy, such as increasing the real cost of servicing foreign denominated debt during a depreciation or reduced export competitiveness from an appreciation. Furthermore, they demonstrate evidence of such practise across a range industrial and emerging economies.

Finally, as noted in the Introduction there has been two main approaches to modelling the possible non-linear relationship. First, the threshold approach that imparts abrupt regime switching such as that considered by Obstfeld and Taylor (1997) and second, the smooth-transition approach that allows for more gradual movement between regimes, examples include Michael et al (1997) and Taylor et al (2001). In this paper we only pursue the latter approach in particular, as argued by Dumas (1992), Michael et al (1997) and Taylor and Sarno (1998) the inherent time aggregation within real exchange rate series and possible non-synchronous trading effects favours the smooth-transition model. Furthermore, the averaging implicit within real exchange rate series, given that the underlying traded goods will have different transaction costs, is suggestive of a smooth rather than discontinuous adjustment process.

3. Data and the Usual Tests.

We use monthly real exchange rate data against the US dollar for Canada, Germany, Japan, Switzerland and the UK. The data on nominal exchange rates and each countries CPI were collected from Datastream over the period 1973:1 to 2006:12, with the exception of Germany were the sample ends in 1998:12. Real exchange rates were calculated in the usual way and summary statistics are not reported as many of these series have been examined in extant work, but are available upon request. Nonetheless, Figure 1 plots the series.

We begin our analysis of the data by providing the usual set of linear unit root tests over the whole sample period. Extant research in the above cited papers typically reports non-stationary behaviour when assessed upon these tests. More specifically, we report in Table 1 the well-known Dickey-Fuller, Phillips-Perron, Kwiatkowski et al unit root tests. Additionally, we also report two tests recently introduced by Ng and Perron (2001) and based on GLS detrending, namely the DF-GLS and the MZà tests. In essence the first test applies the DF testing procedure to GLS detrended data, whilst the second test is a modified Phillips-Perron test again utilising GLS detrending. These two tests have been shown to exhibit good size and power properties, and significantly better properties than the conventional unit root tests (see also Elliot et al, 1996). Finally, we also report the Perron (1997) unit root test with a break; again as is well-known the presence of a break can confound unit root tests and bias them towards the reporting of non-stationarity. These results, in common with those reported elsewhere, indicate non-stationary behaviour in the linear setting, although the Kwiatkowski et al test, which has the null of stationarity, indicates possible stationary behaviour of the real exchange rate for Germany and Switzerland, while the Phillips-Perron test also suggests possible stationarity for Switzerland.

As noted in the introduction the current main research theme in explaining the apparent non-stationary behaviour concerns the presence of possible non-linear dynamics within the series. In particular, the dominant hypothesis is that small deviations from mean value (equilibrium) may exhibit random walk behaviour, as the costs of arbitrage will inevitably outweigh any benefits. However, once equilibrium deviations become large then arbitrage intervention will take place ensuring mean reversion. Furthermore, the greater the deviation from equilibrium the quicker the speed of reversion will be. This belief has motivated the application of the exponential smooth-transition (ESTR) model, and variants thereof. Thus, we proceed to consider a selection of such models, with the results reported in Table 2.

First, we consider the ESTR test of Kapetanios et al (2003, 2006), which has been applied to a series of Yen real exchange rates by Chortareas and Kapetanios (2004). The ESTR test is a simple extension of the usual Dickey-Fuller approach to unit root testing. That is, where the standard Augmented Dickey-Fuller approach is given by:

(1) $$\Delta x_t = \rho x_{t-1} + \varepsilon_t$$

where x_t is the variable of interest and ε_t is a white noise error term, with the null hypothesis of non-stationarity rejected provided ρ is statistically significant and lying in the region $-2 < \rho < 0$. The ESTR testing and modelling procedure extends this approach by allowing a regime of behaviour close to the attractor point of zero to be different from behaviour far away from the attractor. In particular, a restricted version of the ESTR model that can capture the dynamics noted above is given by:

(2) $$\Delta x_t = \rho x_{t-1}(1 - \exp(-\gamma x_{t-1}^2)) + \varepsilon_t$$

where the parameter γ measures the speed of transition between the outer (reverting) and inner (random walk) regimes which switch symmetrically around the attractor point of zero with x_{t-1}. The corresponding ESTR test conducted by Chortareas and Kapetanios (2004) is given by:

(3) $$\Delta x_t = \beta x_{t-1}^3 + \varepsilon_t$$

where should the parameter β be negative and significant then ESTR non-linearity is supported. The results reported by Chortareas and Kapetanios typically support this type of non-linear stationarity.

The ESTR model was also considered by Taylor et al (2001) who estimated an alternate restricted version of the model for a selection of US dollar based real exchange rates. In particular, using the specification below, they reported non-linear reversion in four real exchange rate series:

(4) $$x_t = x_{t-1} - [1 - \exp(-\gamma(x_{t-1} - c)^2)](x_{t-1} - c) + \varepsilon_t$$

where, again, γ measures the speed of reversion from the outer to inner regime, while c determines the equilibrium point.

The ESTR model has generally been preferred over the alternate logistic-STR (LSTR) model as the ESTR model captures symmetric reversion to equilibrium of large deviations. This symmetry is typically preferred in the exchange rate context due to the two-sided nature of the foreign exchange market. The LSTR model, in contrast, is asymmetric between positive and negative deviations. However, several authors have reparameterised the LSTR model to incorporate similar dynamics to those of the ESTR model, i.e. the distinction between small and large deviations and symmetry between positive and negative. One such model is the absolute LSTR (ALSTR) model of Liew et al (2003) who demonstrate good forecasting power on two Yen real exchange rate series. The ALSTR model is given by:

(5) $$\Delta x_t = \rho x_{t-1} (1 + \exp(-\gamma^2 |x_{t-1}|))^{-1} + \varepsilon_t .$$

In a similar vein Sollis et al (2002) introduced the Modified LSTR (MLSTR) model to capture the dynamics of a large selection of US dollar and German mark real exchange rate.

The MLSTR model, like the above ALSTR model, is designed to distinguish between large and small deviation and exhibit symmetric mean reversion. Using the below model Sollis et al reported significant evidence of such mean reversion:

(6) $$\Delta x_t = \rho x_{t-1} (1 + \exp(-\gamma^2 x_{t-1}^2))^{-1} + \varepsilon_t .$$

Finally, we consider the one of the latest additions to this suite of models, namely the Multiple Regime STR (MRSTR) model of Bec et al (2004) and originally proposed by van Dijk et al (2002). In Bec et al's MRLSTR model again the parameters are constrained to ensure symmetric reversion:

(7) $$\Delta x_t = \rho x_{t-1} ((1 + \exp(-\gamma x_{t-1}))^{-1} + (1 + \exp(\gamma x_{t-1}))^{-1}) + \varepsilon_t .$$

It should also be noted that Bec et al do allow for a parameter in the inner regime, although it is nearly always insignificant.

The estimation results from these non-linear alternatives are presented in Table 2 for each of our five real exchange rate series. A point to note is that the usual critical values for rejecting the null are no longer valid in the context of thresholds that are not identified under the null hypothesis of non-stationarity (see, for example, Andrews and Ploberger, 1994; Hansen, 1996). Therefore, in order to obtain usable critical values, where the authors when introducing their non-linear model have derived critical values we utilise those, otherwise we conducted a Monte Carlo experiment whereby 50,000 random walk processes of observation length 500 were generated. The appropriate model was then estimated using this data. The last row in Table 2 presents the 5% critical values. The estimation results from Table 2 suggest that there is no overwhelming evidence of non-linear stationary behaviour within these five real exchange rates. Indeed only the Taylor et al ESTR model for Japan and the MRLSTR model for Switzerland indicate stationarity at the 5% level.

An alternate dynamic process that has been considered within the real exchange rate literature is the presence of fractional integration.[ii] In order to consider this we implement the

Geweke and Porter-Hudak (1983) estimator. To briefly introduce the idea an assumption underlying the standard linear estimation methods is that the parameter of integration, *d*, is equal to zero, in which case the series is stationary and that the effects of a shock die out quickly, or that the integration parameter equals one, *d*=1, in which case the series is non-stationary. However, a more general view is that a series may exhibit long memory following a shock, but be ultimately stationary, such thus, we can allow *d* to take any value between zero and one, and where 0<*d*<0.5 the series is regarded as persistent but stationary. The log-periodogram regression estimator proposed by Geweke and Porter-Hudak (1983; d_{GPH}) is based on the periodogram using the Parzen window. More formally, the spectral density of a time series can be given by:

$$f(\lambda) = \left|1 - \exp(-i\lambda)\right|^{-2d} f^*(\lambda) \tag{8}$$

where *d* is the long memory parameter and $f^*(\lambda)$ represents short-run dependency. Geweke-Porter-Hudak (1983) propose the following estimator based on the first *m* periodogram ordinates:

$$I_j = \frac{1}{2\pi n}\left|\sum_{t=1}^{n} y_t \exp(i\lambda_j t)\right|^2 \quad for\ \ j = 1,\ldots,m \tag{9}$$

where $\lambda_j = 2\pi j / n,\ j=0,1,2,\ldots,m$, defines the set of harmonic frequencies. The least squares regression is thus:

$$\{\log(I_j) : j = 1,\ldots,m\} = \alpha_0 + \alpha_1[\log\left|1 - \exp(-i\lambda_j)\right|] + e_j \tag{10}$$

where $\hat{d} = -(\hat{\alpha}_1 / 2)$.

Following the recent paper by Perron and Qu (2006) we estimated the value of *d* over a range of *m*. In particular, Perron and Qu have argued that doing so will enable us to distinguish between a truly long memory series and a short memory series with occasional breaks. Should the data be characterised by the latter then the value of *d* will decline as *m* increases, on the other hand should the series be truly long-memory than *d* should remain constant. The results, presented in Table 3, suggest a value of *d* close to or indeed above one, again indicating non-stationarity and not stationary fractional integration or structural breaks.

To further illustrate the time-series behaviour of the series and possible non-stationary or long-memory behaviour we also present the full autocorrelation function for each series in Figure 2. Evident from this figure is that each real exchange rate series does appear to exhibit stationarity in that the autocorrelation functions does decay to zero, for example the autocorrelation function for the UK decays to zero after approximately 40 lags. Furthermore, there is evidence of cyclical behaviour within the autocorrelation functions for several of the series, which may in part be the source of the confusing time-series behaviour. Figure 2 also presents the full autocorrelation function for a simulated ESTR model, what is immediately apparent is the similarity between this simulated series and several of our actual real exchange rate series. Such a pattern may arise from possible cycling between two (or more) equilibrium states, so-called limit-cycle behaviour, and where it is well known that such a property is inherent and can be replicated by the ESTR model.

4. A Further Model.

One element the above non-linear models retain in common is the symmetric nature of the reversion to equilibrium. Thus, we aim in this section to consider the possibility of asymmetric reversion to equilibrium in addition to the non-linear characterisation above. More specifically, the previous models allow for a random walk inner regime and a mean-reverting outer regime, however, reversion in this outer regime is symmetric regardless of the starting point of the disequilibrium. However, as noted by Sollis et al (2002) there is no strong *a priori* reason why the factors that govern non-linear behaviour (e.g. transaction costs) may not also generate asymmetric deviations from equilibrium. Thus, we consider that there maybe asymmetry in the speed of reversion between positive and negative deviations or asymmetry in the point to which positive and negative deviations revert. To this end we estimate the following model:

$$(11) \qquad \Delta x_t = \rho_1 x_{t-1} (1-\exp(-\gamma_1 (x_{t-1} - c_1)^2)) I_t + \rho_2 (1-\exp(\gamma_2 (x_{t-1} - c_2)^2))(1 - I_t) + \varepsilon_t$$

where c denotes the threshold value and I is an indicator function depending upon whether the real exchange rate is above or below its mean value.[iii]

The results from estimating this model are presented in Table 4 and present several interesting points. First, each real exchange rate now exhibits non-linear and asymmetric stationarity, to be contrasted with the non-stationarity identified in the above linear and non-linear but symmetric models. In particular, all individual *t*-values are significantly negative while a joint test of significance is likewise statistically different from zero.[iv] Second, that for each series positive and negative deviations from the mean value reverts to different attractor points. Furthermore, these two attractor points are significantly different from each other as evidenced by the test $F_{c1=c2}$.[v] Finally, for three series, Germany, Japan (albeit at the 10% significance level) and Switzerland the speed of mean reversion differs between positive and negative deviations, as given by the test $F_{\rho1=\rho2}$. In contrast, for Canada and the UK the speed of reversion parameters are statistically equal to each other.

5. Discussion of Results.

Further Analysis of Asymmetric-ESTR Model.

The results presented above suggest the general conclusion that observing real exchange rates from the perspective of a linear or non-linear but symmetric model they appear to exhibit non-stationarity. Similarly, if we take a long-memory, fractional integration view, then the same conclusion holds. However, if we take a non-linear and asymmetric viewpoint then we observe stationarity. The results from the linear and symmetric non-linear models sit squarely in the tradition of extant real exchange rate research, namely when taken together with the existing literature cited in Sections 1 and 2, the empirical results are confusing with reported evidence of both stationary and random walk behaviour.[vi] What our results do support is the view that real exchange rates may exhibit both asymmetry and non-linearity, whereas previous studies have typically ruled out asymmetric behaviour within such series. However, as noted by Sollis et al (2002) there is no strong reason why, for example, transaction costs cannot generate both non-linear and asymmetric deviations. Thus the results here add to the small but increasing evidence in favour of asymmetry (see, also Sollis et al, 2002; Leon and Najarian, 2005).

To shed some light upon our results we examine the nature of them from several perspectives. First, we present in Figure 3 ten year rolling *t*-values for the linear, ESTR and MRLSTR models as well as the two *t*-values from the asymmetric-ESTR model presented in Section 3 for the UK. Evident from this figure is that the nature of the stationarity or non-stationarity dynamics within the series has exhibited substantial time-variation. For example, taking the ESTR model we can observe an insignificant *t*-value from the start of the sample until around the mid-1990's. From this point the UK real exchange rate appear to exhibit ESTR stationarity until the early 2000's when the *t*-values became insignificant again, incidentally this period from the mid 1990's is consistent with the results reported by, for example, Taylor et al (2001). Hence, it may be the case that different studies are reporting different behaviour within real exchange rates due to substantial time-variation in their time-series behaviour. With regard to the asymmetric ESTR model presented in Section 3 we can observe a similar general movement of the two *t*-values, although their movement is around a lower value and thus providing greater support for stationarity when we allow for possible asymmetries.

To provide further insight Figure 4 plots the rolling ESTR *t*-values against the UK real exchange rate, from which we can see that when the real exchange rate is rising (depreciating) then it exhibits more non-stationary type behaviour, whilst when the rate is appreciating or fluctuating in no obvious direction then it exhibits more stationary behaviour. Figure 4 also plots the UK real exchange rate against the rolling *t*-values from the asymmetric ESTR model. Here we can observe that when the real exchange rate is rising (depreciating) then the *t*-value for positive deviations is falling, strengthening the statistical significance for stationarity, whilst when the real exchange rate appreciates the *t*-value associated with negative deviations is at its lowest value, again supporting stationarity. Thus, this evidence again demonstrates that allowing for asymmetry in the speed of adjustment and the attractor point can capture the dynamics within the real exchange rate series.

To further analyse our results, following Taylor and Peel (2000) and Dufrenot et al (2006) we report evidence from the transition function with regard to the degree of misalignment and the strength of mean reversion. In particular, Taylor and Peel (2000) introduce two measures based upon the estimated transition functions. First, they define the following function to measure the estimated percentage degree of misvaluation:

$$(12) \qquad G(x_{t-d}) = 100(F(x_{t-d}))\{x_{t-1}/|x_{t-1}|\}$$

where $F(x_{t-d})$ is the estimated transition function. Second, they define as one minus the estimated transition function, $\{1- F(x_{t-d})\}$, as a measure of the degree of mean reversion. Plots for each of our series are presented in Figure 5. These figures present some interesting points that reflect the movement of the real exchange rates over time. To highlight the information in the plots taking Canada we can observe that the real exchange rate has a history of substantial deviations from equilibrium with slow mean reversion, for example, around 1985 there was a persistent overvaluation evidence by the negative values in the lower plot, that demonstrate the overvaluation, and the low values in the upper plot, that demonstrate slow mean reversion. A similar pattern can be found in during the late 1990's and early 2000's. More generally, we can observe in each of the plots the strong appreciation of the dollar during the early to mid 1980's, which is further characterised by persistence and slow mean reversion. While we can also observe the weakening of the dollar rates around the time period covering the late 1980's and early 1990's. Similarly, we can observe the strengthening of the dollar in the early 2000's and its subsequent weakening.

Reasons for Asymmetry.

A final point for discussion concerning our results is as to why there may be asymmetry within real exchange rates. Our model results point to one particular conclusion in that for each rate the estimated speed of reversion is greater for depreciations than for appreciations, albeit for some countries this difference is not statistically significant. Nonetheless, the evidence in Table 4 demonstrates that the absolute value of ρ_1 is greater than the absolute value of ρ_2. This finding of asymmetry is important as many studies rule out the possibility of asymmetric adjustment *a priori*, on the basis of the evidence presented here and in the papers of Sollis et al (2002) and Leon and Najarian (2005) such priors may not hold. Regarding the rationale for such asymmetry Dutta and Leon (2002) and further supported by Leon and Najarian (2005) have argued for a policy intervention based rationale. In particular, they argue that while policy authorities may intervene to curb excessive movement in either direction, there is evidence that depreciations are more vigorously defended.

Furthermore, the asymmetry may arise due to the behaviour of and interaction between different market trader types. In particular, DeGrauwe and Grimaldi (2004) have put forward a behavioural explanation of exchange rate movements and bubbles and crashes arising from the behaviour of fundamental and technical traders, the latter of which utilise extrapolative forecasts to inform trading behaviour. During market rises technical traders by operating feedback trading strategies can start a bandwagon effect, thereby inducing a further round of market rises. The profitability of such extrapolative strategies will encourage more technical traders to enter the market encouraging yet further rises. Fundamental traders while instinctively wishing to trade against the market to restore fundamental value may be discouraged from doing so as the increased weight of technical traders will render such reversion trading unprofitable. A bubble may therefore develop if the weight of technical traders gains sufficient mass such that contrarian fundamental traders decide not to trade (or decide to act as technical traders). Ultimately the market may become dominated by technical traders, at which point the momentum in market rises will diminish allowing fundamental traders to re-establish themselves. A crash can then occur either as a result of some ‘bad’ fundamental news or as technical traders follow extrapolative strategies on the back of fundamental traders actions. This behaviour can therefore lead to persistent deviations from fundamental equilibrium. Furthermore, while in the foreign exchange market bubbles can arise in either direction the degree of persistence may be asymmetric with overvaluations exhibiting greater persistence due to other behavioural characteristics of technical traders in market falls, such that the psychology of market rises versus falls is of importance (for example, it is argued that noise-type traders exhibit more exuberance in overvalued than undervalued markets, see Shleifer, 2000 for a general discussion).

As further support for the possible asymmetry between positive and negative deviations from mean we present the simple ‘deepness’ statistic of Sichel (1993). This statistic measures whether opposite deviations from mean are significantly different from each other and was originally conceived to test for asymmetric business cycle behaviour. The Sichel deepness test is given by:

$$D = \frac{1}{T}\sum_t (x_t - \bar{x}_t)^3 / \sigma(x_t)^3 \qquad (13)$$

where $\bar{x}_t, \sigma(x_t)$ are the sample mean and standard deviation respectively. Given the high probability of autocorrelation within the deepness statistic in order to obtain appropriate standard errors, a series, $z_t = (x_t - \bar{x}_t)^3 / \sigma(x_t)^3$, is generated which is then regressed upon a constant and the deepness test obtained using Newey-West standard errors. Deepness test statistics for each of our series are –2.52, -3.97, -0.26, -2.25 and –1.72 for Canada, Germany, Japan, Switzerland and the UK respectively. These test statistics reveal first that there are all negative indicating that appreciations reach higher peak then depreciations reach lows in support of the behavioural model of DeGrauwe and Grimaldi and second that for three series the test is significant at the 5% significance level (and 10% significant for the UK). This result therefore appears to suggest that asymmetries are potentially important in real exchange rate modelling.

A Forecasting Exercise.

To assess the practical implications of the empirical results reported here we examine the forecasting performance of the asymmetric-ESTR model compared to the existing models within the literature. The forecasting ability of the model is of importance to market traders in devising trading strategies, international portfolio managers in benefiting from the effects of exchange rate fluctuation on their portfolios and finance officers involved in the foreign exchange market for (goods) trading purposes. We conduct two forecasting exercises. First, we compute recursive one-month ahead forecasts for each of our real exchange rate series. Second, we compute recursive one-year (twelve-step) ahead forecasts. In both exercises we estimate each model over the period 1973:1 to 1989:12 and compute the forecast for 1990:1 in the first exercise and the whole of 1990 in the second exercise, we then roll the sample forward one month or one year, re-estimate each model and again compute the monthly or yearly forecast.[vii] In order to assess the relative forecast performance we utilise two measures. First, we consider the Mincer and Zarnowitz (MZ, 1969) regression and a related forecast encompassing test. Specifically, the forecast regression test of predictive power is given by:

(14) $$x_t = \alpha_t + \beta \hat{x}_t + \varepsilon_t$$

where x_t represents the actual value of the real exchange rate and $\hat{x}_t$ represents the forecast values. The forecast encompassing tests, originally considered by Chong and Hendry (1986), allow examination of whether a competing forecast carries additional information over a base model forecast. If the former model carries no additional information then the latter forecast model is said to 'encompass' the former. To test for forecast 'encompassing' we consider the following regression model, which is a basic extension of equation (14).

(15) $$x_t = \alpha + \beta_1 \hat{x}_{1,t} + \beta_2 \hat{x}_{2,t} + \varepsilon_t$$

where the subscripts 1, 2 denote the forecast models (1 refers to the base model and 2 refers to the competing model). The null hypothesis associated with this test is that Model 1 encompasses Model 2 in which case β_2 is equal to zero, whilst if β_2 is greater than zero then Model 2 contains information that Model 1 does not, such that Model 2 is not encompassed by Model 1. In application of the test we use a simple random walk model as the base and consider as alternatives all the models (linear and non-linear) discussed above.

The second forecast performance metric is based upon the usual mean squared error (MSE) approach however, as opposed to reporting the MSE values themselves we consider a test designed to highlight any statistical difference in the reported MSE values. The reason for this, as noted by White (2000), is the possibility that the results are obtained by chance rather than inherent superior performance of a particular model. Therefore we implement the test of superior predictive ability of Hansen (2005). This test evaluates the performance of the alternate models against a benchmark model and assess whether the same outcomes can be obtained from more than one sample by the use of a bootstrap procedure. The details of the test can be found in Hansen (2005) and Hansen and Lunde (2005) but in short where $L(y_t; \hat{y}_t)$ denotes the forecast loss if $\hat{y}_t$ is the prediction and y_t is the realised value. The performance, at time t, of model k relative to a benchmark model, model o, can then be defined as:

(16) $$X_{k,t} = L(y_t; \hat{y}_{o,t}) - L(y_t; \hat{y}_{k,t}).$$

The issue to hand is whether any of the models, $k=1,\dots,K$ are superior to the benchmark model. Hence, we want to test the null hypothesis of $\mu_k = E(X_{k,t}) \leq 0$, as a positive value of μ_k would correspond to model k being superior to the benchmark model. The test statistic is:

(17) $$T_n^{SPA} = \max_k \frac{n^{0.5}\overline{X}_k}{\sigma_k}$$

where $\overline{X}_k = \frac{1}{n}\sum_{t=1}^{n} X_{k,t}$ and $\sigma_k = \mathrm{var}(n^{0.5}\overline{X}_k)$ which is estimated via a bootstrap. The benchmark is the random walk model

The results from the first forecasting metric are presented in Table 5. As noted we conduct both one-step ahead forecasts and twelve-step ahead forecasts. The rationale for such forecast horizons lies with forecast horizons that would be of importance not only to market traders but also to corporate finance managers. The entries in Table 5 are the R^2 from the above regression equation (14) and the t-statistics for β_2 in equation (15), recalling that where β_2 is significantly positive then the base model does not encompass the alternate model. The results from the one-step ahead forecasts suggest that the random walk model has explanatory power similar to the alternate linear and non-linear models, however, for all series except Canada the alternate models are not encompassed by the random walk model, suggesting that there is additional information contained in these models that is not captured by the random walk model.[viii] Finally, it can be noted that the explanatory power of the forecasts is very high for all series, suggesting that one-step ahead real exchange rate series are highly forecastable. Turning to the twelve-step ahead (one-year) forecasts a slightly different picture emerges. First, the random walk model encompasses all other models for Canada, consistent with the one-step ahead forecasts. Second, in contrast to the one-step ahead forecasts the random walk model encompasses the linear and symmetric non-linear models for Germany (except the MRSTR model), Japan and Switzerland, only for the UK does the random walk model not encompass these alternate models. Finally, the asymmetric non-linear ESTR model is not encompassed by the random walk model and provides the highest R^2 for all series, except Canada.

The forecast results based on the MSE measure are presented in Table 6. These forecast results present a similar picture to that in Table 5. The entries in Table 6 are *p*-values of a test designed to determine statistically significant differences in the MSE from the different forecasts model, where the benchmark model is the random walk model. At the one-step ahead horizon the alternate models do not provide significantly smaller MSE values when compared with the random walk MSE. Table 5 provides similar evidence in terms of the closeness of the R^2 values. However, when we turn to the twelve step ahead results we do now see a significant improvement from the random walk benchmark, with the exception of Canada. In sum, these results further support the belief that accounting for possible asymmetries within real exchange rates maybe of importance particularly when forecasting over a longer horizon.

6. Summary and Conclusion.

The time-series properties of real exchange rates remains a contentious issue with the existing literature reporting evidence in favour of stationary and non-stationary dynamics. This paper re-examines the time-series properties of five real exchange rate series in the light of recent research that both supports non-linear stationarity and suggests that such models have limited forecasting power. Furthermore, we also consider possible asymmetries within real exchange rates whereas extant empirical research tends to rule out asymmetric behaviour *a priori*. The chief results presented in this paper show that first the apparent confusing time-series behaviour of real exchange rates arises from the fact that each real exchange rate series exhibits periods of stationary and non-stationary behaviour. That is, the time-series properties of the data exhibit temporal variation. Second, we demonstrate that asymmetries are present within the data and are important in describing the movements of real exchange rates and for obtaining improved forecast notably over a twelve month horizon.

Using real exchange rate data for Canada, Germany, Japan, Switzerland and the UK against the US we test for linear, symmetric non-linear and asymmetric non-linear stationarity. Existing evidence typically rejects linear stationarity and supports symmetric non-linear stationarity, whilst typically ruling out asymmetric behaviour. Using a selection of linear unit root tests and a variety of symmetric smooth-transition based non-linear models we report evidence of non-stationarity, although there is some weak evidence of possible stationarity in one or two series. Using an asymmetric ESTR model in contrast we can report stationary behaviour for all series, suggesting that asymmetries in real exchange rates may be of importance.

To further analyse the behaviour of real exchange rate series and their apparent confusing time-series behaviour we consider evidence of temporal variation in the time-series properties of the data. To this end rolling test statistics demonstrate that real exchange rates exhibit periods of stationary and non-stationary behaviour and such temporal variation may explain the findings of different authors with regard to real exchange rate stationarity. Nonetheless, the asymmetric ESTR model appears to provide the most stable results. Moreover, the behaviour of the asymmetric ESTR model appears to well describe the past dynamic behaviour of real exchange rates.

All the above results suggest that asymmetries within real exchange rates are of importance. To further consider this point and to provide relevance for market participants, such as traders, portfolio managers and finance managers, we conduct a forecasting exercise for real exchange rates over both a month and a year and baseline these results to a simple random walk model. Results suggest that any forecast improvement over the random walk model at the one-month level is at best marginal. At the one-year horizon the asymmetric ESTR model provides significantly better forecasts over all other models (except for Canada for which the random walk model is superior).

In sum, our results suggest that the confusing time-series behaviour of real exchange rates arises as the time-series property of the data differs through time. However, our results do suggest that accounting for asymmetries within real exchange rates may improve the ability to model and forecast such series. What remains is to provide a rationale for such asymmetry. Extant theoretical work argues that policy intervention is one avenue to explain such asymmetries, while another examines the behaviour of fundamental and technical traders. It remains for further research to refine these theoretical approaches and test their prediction on data.

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Table 1. Linear Unit Root Tests						
	ADF	DF-GLS	PP	KPSS	MZa	Perron
Canada	-1.56	-0.74	-1.49	1.31	-1.19	-2.80
Germany	-2.10	-1.75	-1.96	0.26*	-6.07	-4.23
Japan	-2.28	-0.72	-2.44	1.09	-1.24	-4.44
Switzerland	-2.78	-0.81	-2.90*	0.38*	-1.82	-3.80
UK	-2.12	-0.85	-2.17	0.65	-2.39	-3.62
CV	-2.87	-1.94	-2.87	0.46	-8.10	-4.80

Notes: Entries are linear unit root test values, where the row CV presents the 5% critical values. An asterisk denotes stationarity on the basis of the test.

Table 2. Non-Linear Models.						
	ESTR Test	ESTR	ESTR – TPS	ALSTR	MLSTR	MRLSTR
Canada	-1.44	-1.55	-1.31	-0.82	-0.60	-1.46
Germany	-1.56	-2.11	-0.27	-2.07	-1.41	-2.36
Japan	-2.27	-2.22	-2.49*	-1.40	-2.08	-2.34
Switzerland	-2.27	-2.26	-0.57	-2.63	-2.05	-3.03*
UK	-2.45	-2.42	-2.05	-0.91	-0.98	-2.35
CV	-2.93	-2.80	-2.19	-2.87	-3.11	-2.87

Notes: Entries are non-linear unit root test values, where the row CV presents the 5% critical values, see Section 2 for a description of the tests. An asterisk denotes stationarity on the basis of the test.

Table 3. Fractional Integration Estimates.					
Value of *m*	Canada	Germany	Japan	Switzerland	UK
$T^{1/3}$	1.48	1.01	1.10	1.40	0.31
$T^{0.5}$	1.26	1.22	1.06	0.81	0.75
$T^{2/3}$	1.02	1.06	0.97	0.84	0.91
$T^{4/5}$	0.97	0.99	0.91	0.91	0.89
Notes: Entries are estimates of the GPH fractional difference parameter for different values of *m*.					

Table 4. Asymmetric ESTR Models.					
	Canada	Germany	Japan	Switzerland	UK
ρ_1	-0.11 (-3.26)	-0.06 (-5.77)	-0.06 (-4.04)	-0.16 (-3.01)	-0.12 (-2.74)
ρ_2	-0.06 (-3.24)	-0.02 (-3.13)	-0.03 (-4.15)	-0.04 (-3.11)	-0.05 (-3.06)
c_1	-0.10 (-9.03)	-0.39 (-47.11)	-4.61 (-41.82)	-0.25 (-10.75)	0.54 (39.17)
c_2	-0.34 (-26.48)	-0.74 (-29.02)	-5.03 (-68.89)	-0.48 (-7.92)	0.31 (11.03)
γ	5.25 (1.86)	22.43 (1.18)	29.68 (1.09)	16.04 (1.34)	2.44 (2.08)
$F_{\rho1=\rho2=0}$	10.39	21.59	16.82	15.11	10.64
$F_{\rho1=\rho2}$	1.74	25.70	2.76	12.88	2.37
$F_{c1=c2}$	225.18	177.06	177.06	35.52	53.09
Notes: Entries are the estimates with *t*-values in parentheses and coefficient restriction tests of the asymmetric ESTR model presented in Section 4.					

Table 5. Forecast Tests.							
	Models						
	RW	Linear	ESTR	ALSTR	MLSTR	MRSTR	AESTR
	One Step Ahead						
Canada	0.98	0.97 (1.54)	0.97 (0.45)	0.94 (-1.53)	-0.94 (-1.63)	-0.95 (-1.67)	0.98 (0.50)
Germany	0.89	0.91 (3.69)	0.91 (3.71)	0.91 (3.70)	0.91 (3.71)	0.91 (3.62)	0.92 (5.04)
Japan	0.96	0.97 (4.51)	0.97 (4.51)	0.98 (4.52)	0.97 (4.51)	0.97 (4.63)	0.97 (4.55)
Switzerland	0.95	0.95 (3.05)	0.95 (2.23)	0.95 (2.96)	0.96 (4.29)	0.95 (3.05)	0.95 (4.88)
UK	0.89	0.93 (11.57)	0.93 (11.59)	0.94 (11.58)	0.87 (-2.48)	0.94 (11.58)	0.93 (10.56)
	Twelve Step Ahead						
Canada	0.88	0.87 (-3.31)	0.74 (-2.89)	0.75 (-0.95)	0.77 (-3.31)	0.77 (-0.75)	0.82 (0.45)
Germany	0.22	0.23 (1.53)	0.23 (1.56)	0.23 (1.53)	0.24 (1.85)	0.32 (4.25)	0.62 (10.50)
Japan	0.73	0.70 (-0.82)	0.70 (-0.82)	0.69 (-0.72)	0.70 (-0.82)	0.72 (0.69)	0.75 (5.09)
Switzerland	0.60	0.58 (0.11)	0.60 (1.89)	0.58 (0.09)	0.56 (-0.30)	0.58 (0.11)	0.67 (6.81)
UK	0.53	0.54 (2.31)	0.54 (3.17)	0.54 (2.31)	0.53 (2.02)	0.58 (2.31)	0.60 (7.31)
Notes: Entries are the R^2 from the forecast regression test, equation (14), and the *t*-statistics for the encompassing test, β_2 in equation (15).							

Table 6. Tests of Superior Predictive Accuracy.				
Canada	Germany	Japan	Switzerland	UK
One Step Ahead				
0.74 (0.54, 0.74)	0.54 (0.54, 0.74)	0.56 (0.46, 0.56)	0.46 (0.27, 0.46)	0.27 (0.27, 0.54)
Twelve Step Ahead				
0.46 (0.27, 0.46)	0.00 (0.00, 0.00)	0.00 (0.00, 0.04)	0.00 (0.00, 0.00)	0.00 (0.00, 0.00)

Notes: Entries are p-values for the Hansen (2005) test of superior predictive accuracy. A significant p-value indicates that on the basis of the MSE forecast metric, the benchmark (random walk) model is outperformed by an alternate model. The numbers in parenthesis are the lower and upper bounds for the p-values.

Figure 1. Real Exchange Rate Series

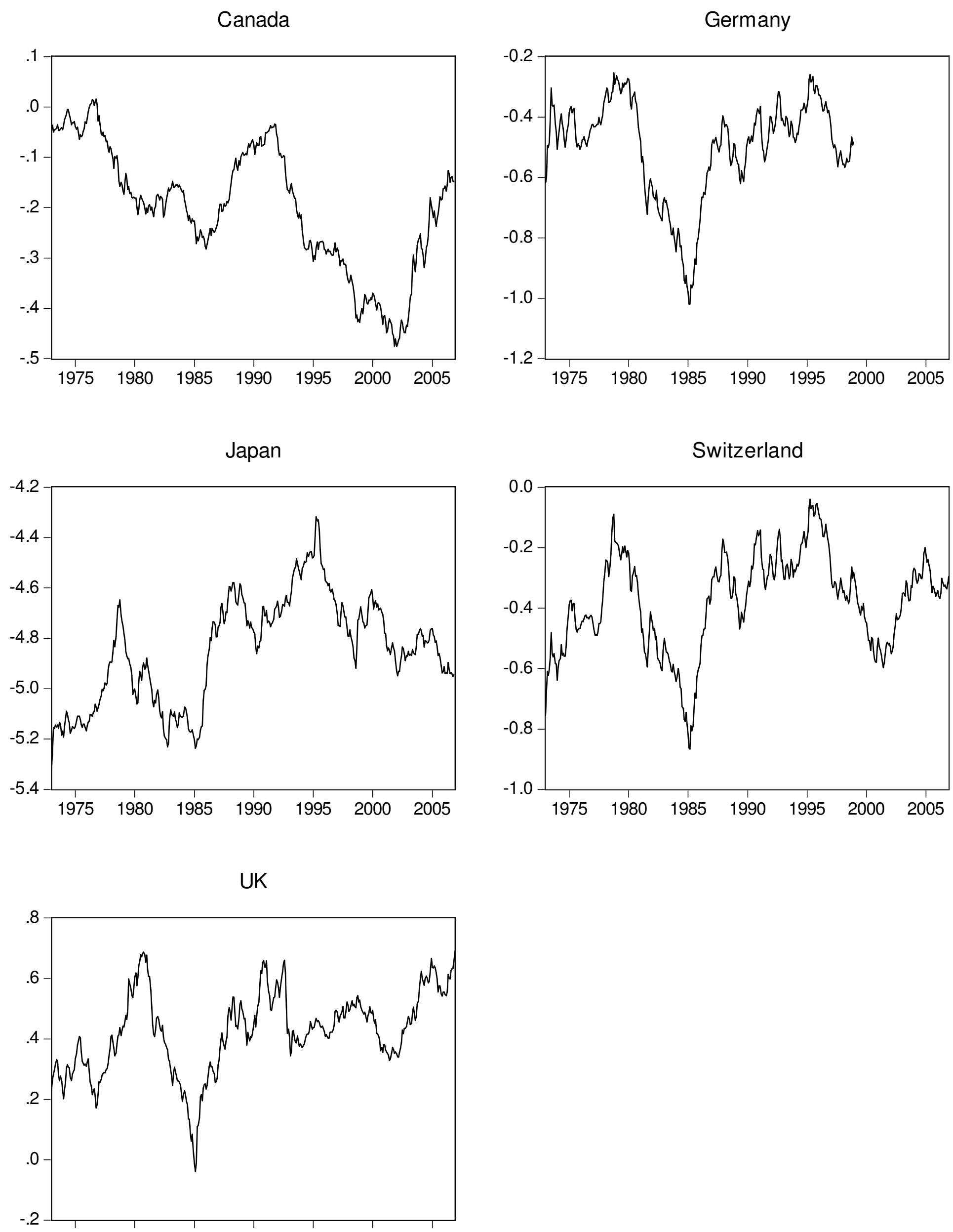


Figure 2. Autocorrelation Functions.

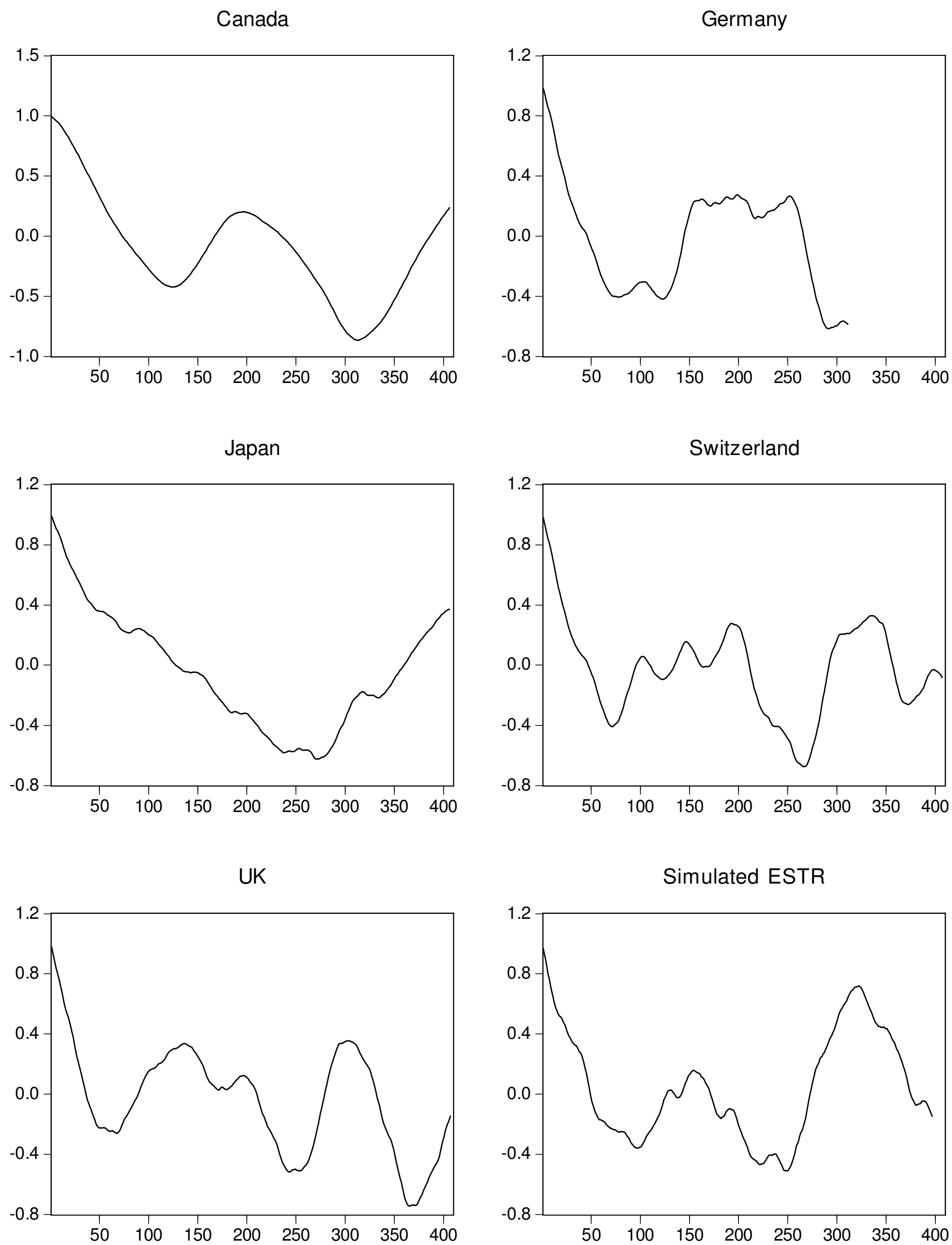


Figure 3. Rolling t-statistics - UK

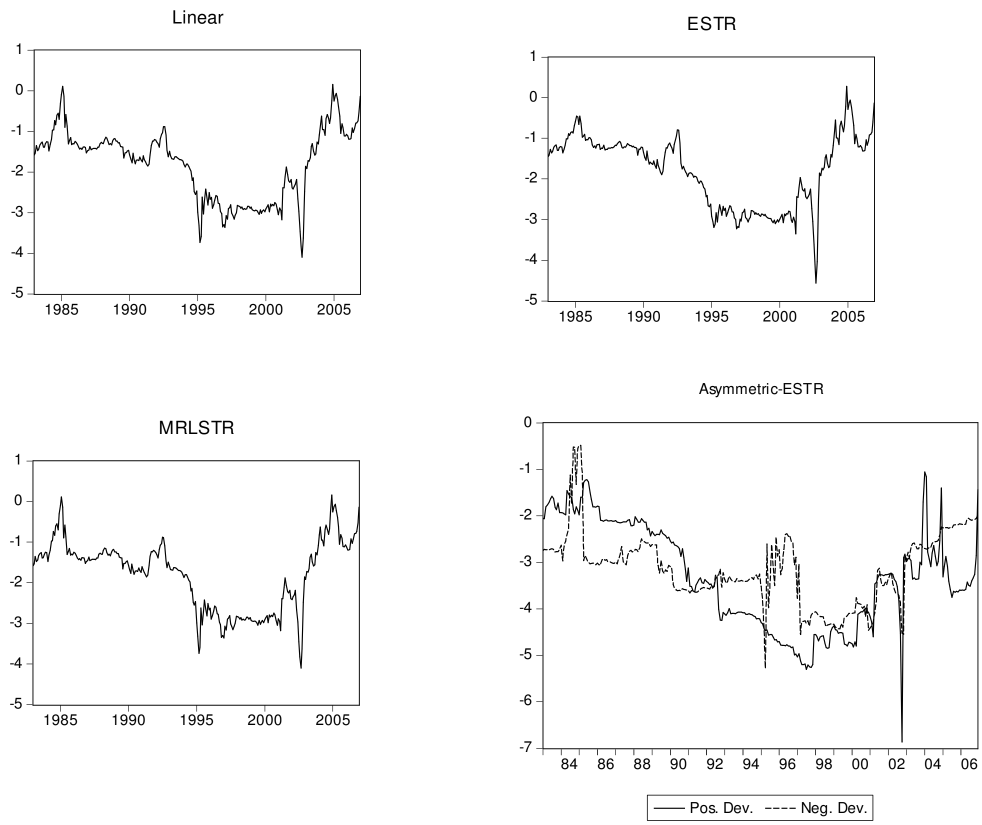


Figure 4. Rolling t-stats and Real Exchange Rate - UK

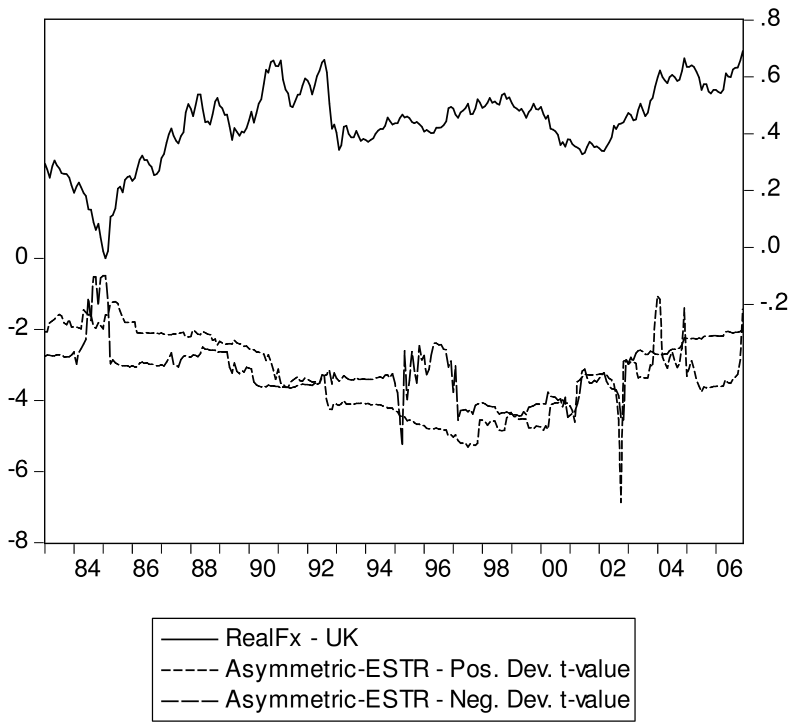
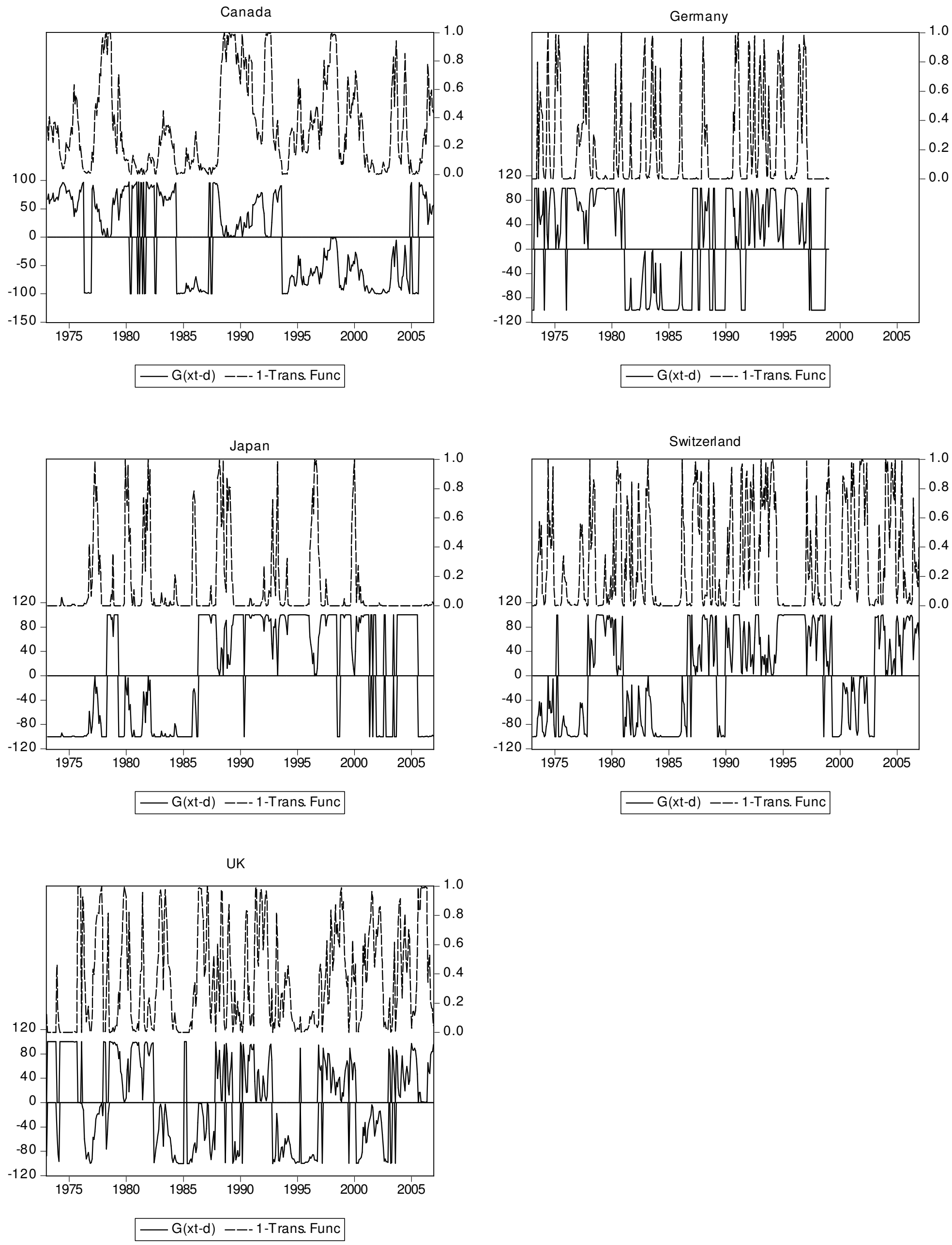


Figure 5. Misvaluation and Mean Reversion.



Notes

[i] Such a conclusion could point to inherent temporal instabilities in the real exchange rate data, such that any one set of estimates could be sample specific or subject to a White (2000) type data-snooping argument.

[ii] See for example, Masih and Masih (2004) and Villeneuve and Handa (2006).

[iii] Infact, in order to establish an equilibrium position, in addition to using the mean value of the real exchange rate, we also estimate the cointegrating equation between the nominal exchange rate and the relative price levels and use the deviations captured by the error-correction term to determine positive and negative deviations. Results are similar across both definitions.

[iv] In order to test for global stationarity we use the test $F_{\rho 1=\rho 2=0}$. As noted with respect to the tests statistics presented in Table 2 we cannot rely on the usual *F*-distribution but can obtain usable critical values via a simulation exercise. Using a similar experiment to that described in Section 3, the 5% critical value is approximately 6.

[v] For which the usual *F*-statistic critical value can be used.

[vi] Indeed the results presented here suggest a mix of linear and symmetric non-linear stationary and non-stationarity across even these five series.

[vii] The yearly, or twelve-step, ahead forecasts involve generating multi-step forecasts. As is well-know however, multi-step forecasting of non-linear processes depends upon the assumptions made about the error term. More specifically, where the one-step ahead forecast is given by (for simplicity in an AR(1) framework): $x^f_{t+1} = E(x_{t+1}|I_t) = G(x_t;\ \beta)$ where $G(.)$ is the non-linear function and E is the expected value, conditional on the information set at time t, I_t. The second-step head forecast is given by: $x^f_{t+2} = E(x_{t+2}|I_t) = E(G(x_{t+1};\ \beta\ |I_t)$. Given that linear and non-linear conditional expectations cannot be interchanged, that is $E(G(.)) \neq G(E(.))$, the relationship between the one-step ahead and two-step ahead forecasts is given by: $x^f_{t+2} = E(G(G(x_t;\ \beta) + v_{t+1};\ \beta)\ |I_t) = E(G(x^f_{t+1} + v_{t+1};\ \beta)\ |I_t)$. In order to obtain multi-step forecasts we therefore approximate the conditional expectation in $x^f_{t+2} = E(G(G(x_t;\ \beta) + v_{t+1};\ \beta)\ |I_t) = E(G(x^f_{t+1} + v_{t+1};\ \beta)\ |I_t)$ through the use of Monte Carlo simulation. That is, the two-step ahead Monte Carlo forecast is given by: $x^f_{t+2} = 1/k\ \Sigma^k_{i=1}\ G(x^f_{t+1} + v_i;\ \beta)\ |I_t)$, where k is the number of repititions. Previous research presented by Lin and Granger (1994) and Clements and Smith (1997) has suggested that the Monte Carlo approach in obtaining multi-step ahead forecasts in a non-linear setting is favourable compared to alternate approaches which attempt to derive the conditional expectation directly (see also, Brown and Mariano, 1989).

[viii] Although not reported, but available, it can be noted that where the linear mean reverting model is the base model then the alternate non-linear models are generally encompassed by the linear at the one-step ahead forecast horizon. This suggests that at this horizon the linear model suffices in obtaining real exchange rate forecasts.