Sectoral Price Changes and Output Growth: Supply and Demand in General Equilibrium[*]

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Abstract: Price changes and output growth, both at the aggregate and the sectoral level, appear to be negatively correlated. At a basic level, this suggests that sectoral "supply" shocks are more prevalent than sectoral "demand" shocks. However, it is not clear what these sectoral price-output correlations mean once one thinks in terms of general equilibrium. To help us understand the implication of these price-output correlations, this paper examines a multi-sector dynamic general equilibrium model that includes sectoral technology shocks and sectoral demand shocks, as well as aggregate money growth shocks. We show that while a model driven solely by sectoral technology shocks can generate "plausible" price-output correlations, "demand" shocks, particularly sectoral demand shocks, are needed for the model to generate the sectoral price-output correlations observed in the data. We also show that technology shocks do not always look like "supply" shocks. Positive technology shocks to sectors producing goods that are used for investment frequently result in increases in output and prices in other sectors while positive technology shocks to sectors producing goods that are used primarily as intermediate inputs look like supply shocks in other sectors.

Keywords: Mulisector Dynamic General Equilibrium Model; Sectoral Technology Shocks; Sectoral "Demand" Shocks

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1. Introduction

The relationship between price and output has been a central theme of the business cycle research. Much of this literature has focused on the Phillips Curve, which emphasizes a positive relationship between inflation and the level of economic activity (see King and Watson 1994, Gordon 1997, Stock and Watson 1999). Recently, several studies have questioned whether prices were indeed procyclical. Cooley and Ohanian (1991) find little evidence in over a hundred years of data of a consistent relationship between output and price in the United States. Cross-country studies such as Backus and Kehoe (1992), Fiorito and Kolintzas (1994), and den Haan and Summer (2001) suggest that countercyclical price movement is quite common. Balke and Wynne(2001) document that a negative relationship between output and prices predominates at the sectoral level as well. They examine NIPA measures of sectoral price and output as well as prices and output for four digit SIC level manufacturing industries. Thus, countercyclical price movement appears to be the rule rather than the exception.[1]

In this paper, we examine further the relationship between price and output at the sectoral level. As in Balke and Wynne (2001), we find a negative relationship between sectoral prices and output. In addition, we show that the correlation between price changes and sectoral Solow residuals is also negative and is arguably stronger than the relationship between sectoral output growth and price changes. A simple economic interpretation of the negative relationship between sectoral price and output is that

[1] A related literature has examined the relationship between inflation and the distribution of relative price changes. In particular, Vining and Elwertowski (1976) have examined the relationship between inflation and the standard deviation and skewness of the distribution of relative price changes for the U.S. while Domberger (1987) has done likewise for the U.K. Ball and Mankiw (1995) and Balke and Wynne (2000) provide alternative theoretical explanations for the existence of such relationships.

sectoral supply shocks are important sources of sectoral fluctuations. However, as we show below, the fact that the price-output relationship is weaker than the price-Solow residual relationship suggests that sectoral demand shocks could play an important role in the behavior of sectoral output and prices.

The problem with thinking about sectoral prices and output in terms of sectoral demands and supplies is that in a general equilibrium framework it is hard to label what is a demand shock and what is a supply shock. As a result, we build a simple multisector dynamic general equilibrium model to help us evaluate what factors might be important contributors to the price-output correlations that we see in the data. While several multisector models have appeared in the literature recently (see Huffman and Wynne 1999, Horvath 2000, Balke and Wynne 2000), none has examined in detail the price-output relationship implied by those models.

We find that both sectoral technology shocks and shocks to sectoral autonomous spending (for example, sectoral government expenditure shocks) are needed for our model to generate sectoral price-output correlations close to those observed. In addition, we show that technology shocks do not always look like "supply" shocks. That is, positive technology shocks in sectors whose output is used primarily for investment and consumption have effects on other sectors that are reminiscent of demand shocks: price and output rise. However, they begin to look more like supply shocks at longer horizons. On the other hand, positive technology shocks to sectors whose output is used primarily as intermediate inputs do indeed have effects on other sectors that look like the effect of a traditional supply shock: prices fall and output rises.

The paper is organized as follows. In the next section, we review the relationship between sectoral price, sectoral output, and sectoral Solow residual. In section 3, we present a simple multisector dynamic general equilibrium model with a cash-in-advance constraint. We will use this model to help us understand the economic significance of the negative sectoral price/output correlation apparent in the data. In section 4, we discuss how we calibrate the parameter values and various types of shocks for the model. In section 5, we examine which features of the model, the type of shocks and the nature of sectoral interactions, have important effects on sectoral price-output correlations while in section 6, we examine how sectoral output and prices respond to various types of sectoral shocks. Section 7 concludes.

2. The Relationship between Sectoral Price, Output, and Solow Residual

Cooley and Ohanian (1991) show that contrary to the widely held view, procyclical behavior of prices is an exception rather than a rule. Only during the inter-war period was the price level unequivocally procyclical. Using quarterly data for the entire post-war period from 1948 to 1987, they find a negative correlation between log-differenced output and prices, that varies between –0.05 and –0.38. Estimates for the sub-sample 1966:1-1987:2 show the strongest negative relationship. Other methods of detrending the data, such as linear detrending or Hodrick-Prescott filtering, yield even stronger results. Balke and Wynne (2001) show that a negative correlation between prices and output is widespread at the sectoral level as well, using 2 digit SIC level NIPA data and 4 digit SIC level data for manufacturing.

Here, we examine the relationship between price and output at the sectoral level using a different data set for the post-war U.S. economy. The primary source of data for our analysis is an extended version of the KLEM data set that consists of annual observations on gross output, price and various inputs for 35 sectors of the US economy, originally compiled by Jorgenson, Gollop and Fraumeni (1987). This data set covers the period from 1947 to 1989 and these thirty-five sectors roughly match the 2-digit SIC of the U.S. industries. This data set forms the basis for the calibration of sectoral technology shocks used in our dynamic general equilibrium model below.

Table 1 presents summary statistics of correlation of sectoral price changes (log first differences) with sectoral output growth, between price changes and sectoral Solow residual growth[2], and of output growth with Solow residual growth. The first column presents average correlations between these variables, averaged over thirty- five sectors. Median and standard deviation of these sectoral correlations are shown in the second and the third column. The fourth and the fifth column present the minimum and the maximum of these correlations across sectors.

As we can see, on average price changes and output growth are negatively correlated. However, price-output correlation varies across sectors, ranging from –0.60 for 'electric utility services' to 0.33 for 'lumber and wood products'. For most of these thirty-five sectors the correlation is negative and only for four sectors it is positive.

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[2] Using the KLEM data set, we calculate the growth of standard Solow residuals for each sector , i.e. $\log(A_{i,t}) - \log(A_{i,t-1}) = [\log(Y_{i,t})- \log(Y_{i,t-1})] - \alpha_K[\log(K_{i,t})-\log(K_{i,t-1})]-\alpha_H[\log(H_{i,t})-\log(H_{i,t-1})] - \alpha_N[\log(N_{i,t})-\log(N_{i,t-1})]$where $A_{i,t}$ is a measure of productivity (technology) in sector i in period t, $Y_{i,t}$ is gross output in sector i at period t, $K_{i,t}$ is the net stock of capital, $H_{i,t}$ is the labor input and $N_{i,t}$ is the material inputs. α_K, α_H, and α_N are income shares of capital, labor, and material inputs respectively. In recent years, researchers have come up with several modifications of the standard Solow residual. These modifications include adjustments for increasing returns to scale or for cyclical variations in input uses (for example, see Hall 1990 and Basu and Kimball 1997). However, the signs of the correlations considered here are robust to all these alternative measures of Solow residual.

Furthermore, the correlation between price changes and Solow residual growth is negative, and larger in magnitude than the ones that exist between prices and output, and between output and Solow residuals.

To help understand the observed relationship, and to explain the motivation behind our analysis let us consider a partial equilibrium model in which demand and supply interact to set equilibrium price and quantity for each sector. Under the assumption of constant returns to scale technology and a fixed vector of input prices, we have horizontal supply curve for sector i (S_i in Figure 1(a) and 1(b)). Note that this supply function, $q^s_i(\mathbf{P})$, is homogenous of degree zero in input and output price vector, **P**. Demand for good i is given by a downward sloping curve, D_i. The demand function, $q^d_i(\mathbf{P},\mathbf{NY})$ is homogenous of degree zero in the price vector, **P**, and nominal income, **NY**. Now suppose sector i experiences a positive technology shock. As in Figure 1(a), the supply curve shifts downward: price falls and output rises. In this partial equilibrium setting, this technology shock does not have any effect on demand. Thus, if there were only technology shocks, one would expect changes in price and changes in output to be perfectly negatively correlated. Similarly, changes in price and changes in technology would be perfectly negatively correlated. However, a purely nominal shock, on the other hand, would shift both supply and demand curves upward by the same amount resulting only in an increase in price (see Figure 1(b)). Thus, purely nominal shocks would tend to lower both the correlation of price changes with output growth, and the correlation of price changes with Solow residual growth. However, if there were an increase in sectoral demand, sectoral output could rise (but not sectoral price) weakening the correlation between sectoral output and prices but not between sectoral prices and sectoral Solow residuals. In other words, the correlation between sectoral output and sectoral Solow residuals falls in the presence of sectoral demand shocks. The fact that we observe a smaller correlation between output and Solow residual growth than between price and Solow residual growth (in absolute terms) suggests that sectoral demand shocks could be important source of relative movements of sectoral price and output.

However, once we move away from a partial equilibrium framework, it is much more difficult to think in terms of supply or demand shocks. For example, if sectors interact with one another through input-output structures, a technology shock in one sector would change demand and supply conditions in other sectors. For example, if good i is used as an input, then positive technology shock in sector i would cause the average cost to fall in other sectors and thus their supply curves would shift downward. Alternatively, a technology shock in a given sector could change demand in other sectors as firms and consumers respond to changes in relative prices. As a result, it is not clear what will be the final effects on prices and output across sectors of a positive technology shock in a given sector.

In order to understand the behavior of sectoral prices and output in a general equilibrium setting with various forces of demand and supply working at tandem, we need a general equilibrium model, to which we now turn.

3. A Simple Multisector General Equilibrium Model

In this section we present a multisector dynamic model with capital accumulation and a cash-in-advance constraint. While there have been recent attempts to build sectoral general equilibrium models (Long and Plosser 1983, Huffman and Wynne 1999, Horvath

2000, Balke and Wynne 2000), with the exception of Balke and Wynne (2000), these models have not examined the behavior of sectoral prices. The current model differs from that in Balke and Wynne (2000) by including capital accumulation and a cash-in-advance constraint for consumption goods. In Balke and Wynne, money only affects the aggregate price level. In the model presented below, with a cash-in-advance constraint, money growth shocks will now have relative price implications as well as aggregate price level implications.

3.1. Economic Environment

There are J different sectors in the economy, each producing a different good. Each sector consists of a large number of identical firms. Each firm has constant returns to scale technology that uses capital, labor and material inputs. There are potentially four different uses for the output of each sector: private consumption, investment input, intermediate inputs in other sectors, and government (or more generally, autonomous) purchases.

A. Consumers and Preferences

We assume that the economy is populated by a large number of identical infinitely-lived consumers. The representative consumer has time-separable preferences summarized by the following utility function:

$$U = \sum_{t=0}^{\infty} \beta^t \left(\sum_{j=1}^{J} \theta_j \log\left(c_{j,t}\right) + \theta_0 \log\left(1 - \sum_{j=1}^{J} h_{j,t}\right)\right) \tag{1}$$

where $0 < \beta < 1$ is the discount factor and $\theta_j \geq 0$ for $j = 1,2,\ldots\ldots J$. $c_{j,t}$ is the consumption of the jth commodity in period t, while $h_{j,t}$ is the hours supplied to the jth production activity in period t. We normalize the time endowment to unity.

The consumer earns wage income by supplying labor to various production activities and rental income by renting out capital goods. She also receives a nominal lump-sum transfer T_t from the government in each period t. Besides consumption, she purchases investment goods for J different sectors to augment her stock of capital goods. Thus the budget constraint for the representative consumer is given by

$$\sum_{j=1}^{J} W_{j,t} h_{j,t} + \sum_{j=1}^{J} R_{j,t} k_{j,t} + \sum_{j=1}^{J} \pi_{j,t} + T_t + m_{t-1} \geq \sum_{j=1}^{J} P_{j,t} c_{j,t} + \sum_{j=1}^{J} Q_{j,t} z_{j,t} + \tau_t + m_t \tag{2}$$

where $W_{j,t}$ is the nominal wage in sector j in period t; $R_{j,t}$ is the nominal rent paid by sector j in period t and $k_{j,t}$ is the capital rented out to sector j. m_t is the nominal money balance with the consumer at the end of the period t. $P_{j,t}$ is the price of commodity j in period t. $Q_{j,t}$ is the price of capital in sector j and $z_{j,t}$ is the gross investment of the representative consumer in sector j in period t. τ_t is the lump-sum tax paid by the consumer while T_t is a lump-sum money transfer in period t. $\pi_{j,t}$ is redistributed profits of (the representative) firm in sector j. We assume that labor is perfectly mobile across sectors. This will imply that in equilibrium $W_{j,t} = W_t$.

The representative consumer also faces a cash-in-advance constraint given by

$$m_{t-1} + T_t \geq \sum_{j=1}^{J} P_{j,t} c_{j,t} \tag{3}$$

The primary purpose of introducing a cash-in-advance constraint is to enable our model to produce nominal prices. Because not all goods are subject to the cash-in-advance constraint, monetary shocks will have implications for relative prices.

The stock of capital in sector j evolves according to

$$k_{j,t+1} = (1 - \delta_j) k_{j,t} + z_{j,t} \tag{4}$$

where $z_{j,t}$ is the gross investment in sector j during period t and δ_j is the rate of its depreciation. Substituting (4) and $W_{j,t} = W_t$ into (2), we obtain

$$\sum_{j=1}^{J} W_t h_{j,t} + \sum_{j=1}^{J} R_{j,t} k_{j,t} + \sum_{j=1}^{J} \pi_{j,t} + T_t + m_{t-1} \geq \sum_{j=1}^{J} P_{j,t} c_{j,t} + \sum_{j=1}^{J} Q_{j,t} k_{j,t+1} - \sum_{j=1}^{J} Q_{j,t} (1-\delta_j) k_{j,t} + \tau_t + m_t \tag{5}$$

The problem faced by the representative consumer is to maximize her discounted utility stream given by (1) subject to the budget constraint (5) and the cash-in-advance constraint (3).

B. Firms and Production

The technology available to a representative firm in sector i is given by a constant returns to scale, Cobb-Douglas production function:

$$y_{i,t} = A_{i,t} (\mu_{i,t} h_{i,t})^{b_i} k_{i,t}^{\alpha_i} \prod_{j=1}^{J} n_{i,j,t}^{a_{i,j}} \tag{6}$$

where $b_i > 0$, $\alpha_i > 0$ and $b_i + \alpha_i + \sum_{j=1}^{J} a_{i,j} = 1$. $y_{i,t}$ is the gross output of sector i in period t; $A_{i,t}$ denotes total factor productivity in sector i in period t; $\mu_{i,t}$ denotes labor augmenting technical change in sector i in period t; $h_{i,t}$ and $k_{i,t}$ are respectively the labor input and the capital stock in sector i in period t; and $n_{i,j,t}$ is the quantity of commodity j used as an input in sector i in period t. The firm maximizes its profits subject to a constraint imposed by the technology as given by (6) in each period. The profit function for firm i in period t is given by

$$\Pi_{i,t} = P_{i,t} y_{i,t} - W_t h_{i,t} - R_{i,t} k_{i,t} - \sum_{j=1}^{J} P_{j,t} n_{i,j,t} \tag{7}$$

where $P_{i,t}$ is the price of output in sector i (or good i).

C. Investment Good Producer

There is an investment good producer who combines goods (but uses no additional capital or labor) to produce investment goods according to the following technology:

$$\sum_{i=1}^{J} z_{i,t} = \prod_{j=1}^{J} x_{j,t}^{d_j} \tag{8}$$

where $x_{j,t}$ is the quantity of commodity j used to assemble investment good. The problem faced by this producer is to maximize profits subject to the constraint (8) in each period. Profit of the investment good producer is given by

$$\sum_{i=1}^{J} Q_{i,t} z_{i,t} - \sum_{i=1}^{J} P_{i,t} x_{i,t} \tag{9}$$

Note that the specifications of (8) and (9) imply that it is costless to move capital from one sector to another.

The reason for having an investment or capital good producer is that the input-output tables are specific about the commodities used as intermediate inputs, but do not provide any breakdown of different types of capital used by a particular industry (sector). The input-output table, however, does tell the amount of a commodity used for investment, denoted by $x_{j,t}$ in our model. Thus, in the input-output tables there is just one type of investment good, but many commodities are used to form that investment good. By introducing investment good producer, we retain that structure in our model.

D. Government

Sectoral government expenditures are assumed to be exogenous. However, government faces the following budget constraint:

$$T_t + \sum_{i=1}^{J} P_{i,t} g_{i,t} = \tau_t + M_t - M_{t-1} \tag{10}$$

We assume that lump-sum transfer T_t is equal to change in stock of money, M_t - M_{t-1}, where M_t is the per capita money supply in period t. The money stock follows a law of motion

$$M_t = \nu_t M_{t-1}, \tag{11}$$

where ν_t is the gross growth rate of the money supply in period t.

E. Sectoral Resource Constraints

Finally, for each sector i, i=1,2,…J, total uses of the commodity must not exceed output. That is,

$$y_{i,t} = c_{i,t} + x_{i,t} + g_{i,t} + \sum_{j=1}^{J} n_{j,i,t} \tag{12}$$

Since we cannot obtain time series data on sector specific government spending, we use time series on sectoral government spending plus sectoral net exports in our calibration and simulation. Thus, $g_{i,t}$ will actually correspond to sectoral autonomous expenditures.

3.2. Steady State Growth

Following King, Plosser and Rebelo (1987, 1988), we have restricted preferences and technologies so that the system exhibits balanced growth. In particular, we assume that the uses of sector i's output given in equation (12) exhibit balanced growth. The model also assumes the growth rate of work effort to be zero, i.e., $\eta_{h_i} = 1$, i=1,2,…J where $\eta_{h_i} = \frac{h_{i,t+1}}{h_{i,t}}$ for all t, is the steady state gross growth rate of work effort.

Let η_{ω_i} be the steady state gross growth rate of variable ω in sector i, then balanced growth of the uses of sectoral output requires that

$$\eta_{y_i} = \eta_{c_i} = \eta_{x_i} = \eta_{g_i} = \eta_{n_{j,i}} \tag{13}$$

Let η_{μ_i} be the growth factor associated with labor augmenting technological progress for production in sector i. Note that the uses of output in sector i (see Equation 12) need not be the growth factor for output in sector i.

Also, balanced growth in the allocation of investment goods across sectors implies $\eta_{z_i} = \eta_z$. Thus, the growth factor for the investment good producer is

$$\eta_z = \prod_{j=1}^{J} \eta_{x_j}^{d_j} \tag{14}$$

The capital accumulation technology, however, implies

$$\eta_{k_j} = \eta_{z_j} \tag{15}$$

which along with (13) and (14) implies

$$\eta_{k_j} = \eta_k = \prod_{j=1}^{J} \eta_{y_j}^{d_j} \tag{16}$$

Combining equations (13) and (16) and the sectoral production functions (Equation 6), we obtain

$$\mathbf{log(\eta_Y) = (I - \alpha d' - a)^{-1} b \, log(\eta_\mu)} \tag{17}$$

where $\mathbf{log(\eta_Y)}$ is a J × 1 vector of logarithm of the growth factor of sectoral output, **I** is a J × J identity matrix, α is a J × 1 vector of α_i's, **d** is a J × 1 vector of d_i's, **a** is the J × J input-output matrix, **b** is a J × J diagonal matrix of b_i's and $\mathbf{log(\eta_\mu)}$ is a J × 1 vector of logarithm of the growth factors of labor augmenting technological progress in J sectors. From (16) it follows that

$$\mathbf{log(\eta_k) = d'log(\eta_Y)} \tag{18}$$

where $\mathbf{log(\eta_k)}$ is the logarithm of the growth rates of sectoral capital stocks.

In our analysis below, we normalize real sectoral output and sectoral demand components by their respective sectoral growth components $D_{i,t} = (\eta_{y_i})^t$.Sectoral capital is normalized by the growth factor for capital $D_{k,t}$ where $D_{k,t} = (\eta_k)^t = \prod_{j=1}^{J} D_{j,t}^{d_j}$. So that nominal expenditures are constant in the steady state, we normalize nominal sectoral expenditures by M_t. Thus, the normalized price of sectoral output is $p_{i,t} = P_{i,t} D_{i,t}/M_t$; the normalized price of sectoral capital is $q_{i,t} = Q_{i,t} D_{k,t}/ M_t$; the normalized wage rate is $w_t = W_t / M_t$; and the rental rate on sectoral capital is $r_{i,t} = R_{i,t} D_{k,t} / M_t$. Finally, both money holdings and lump sum taxes are normalized by M_t.

3.3. Decentralized Optimization

A. First-order Conditions for the Representative Agent's Utility Maximization

Recall that the representative agent's utility function is given by equation (1), her budget constraint is given by equation (5) and the cash-in-advance constraint is given by equation (3). Deriving and normalizing the first order conditions for the consumer's utility maximization yields[3]:

$$\frac{\theta_i}{c_{i,t}} = p_{i,t}(\lambda_t^* + \gamma_t^*) \tag{19}$$

[3] Note that the variables are normalized; for parsimony we are not using different notation.

$$\frac{\theta_0}{1-\sum_{j=1}^{J} h_{j,t}} = \lambda_t^* w_t \tag{20}$$

$$E_t\left[\beta\lambda_{t+1}^*\left[\frac{r_{i,t+1}}{q_{i,t+1}} + (1-\delta_i)\right]\frac{q_{i,t+1}}{q_{i,t}}\right] = \lambda_t^*\eta_k \tag{21}$$

$$\lambda_t^* = E_t\left[\frac{\beta}{\nu_{t+1}}(\lambda_{t+1}^* + \gamma_{t+1}^*)\right] \tag{22}$$

$$E_t\left[\begin{array}{l}\sum_{j=1}^{J} w_t h_{j,t} + \sum_{j=1}^{J} r_{j,t}\, k_{j,t} - \sum_{j=1}^{J} p_{j,t}\, c_{j,t} - \sum_{j=1}^{J} q_{j,t}\, k_{j,t+1}\eta_k + \\ \sum_{j=1}^{J}(1-\delta_j) q_{j,t}\, k_{j,t} + \frac{1}{\nu_t} m_{t-1} + \frac{\nu_t - 1}{\nu_t} - \tau_t - m_t\end{array}\right] = 0 \tag{23}$$

$$\frac{m_{t-1}}{\nu_t} + \frac{(\nu_t - 1)}{\nu_t} - \sum_{j=1}^{J} p_{j,t}\, c_{j,t} = 0 \tag{24}$$

Finally, $\lim_{t\to\infty}\beta^t\lambda_t^* q_{i,t} k_{i,t+1} = 0$, i=1,2,….J and $\lim_{t\to\infty}\beta^t\lambda_t^* m_t = 0$ are the transversality conditions for sectoral capital and money, respectively. The variables λ_t and γ_t are the Lagrange multipliers for the budget constraint and the cash-in-advance constraint in the consumer's problem, respectively. For convenience, we make the following transformations:

$\lambda_t^* = \lambda_t / \beta^t$ and $\gamma_t^* = \gamma_t / \beta^t$.

B. First-order Conditions for Sectoral Output and Investment Good Producers

Sectoral output producers maximize profits (Equation 7) given production technology (Equation 6) and output and input prices yielding:

$$b_i \, p_{i,t} \, y_{i,t} = w_t h_{i,t} \tag{25}$$

$$\alpha_i \, p_{i,t} \, y_{i,t} = r_{i,t} k_{i,t} \tag{26}$$

$$a_{i,j} p_{i,t} \, y_{i,t} = p_{j,t} n_{i,j,t} \tag{27}$$

Note that given the Cobb-Douglas production technology, the input cost shares are equal to their respective output elasticities.

The first- order conditions for profit maximization by the producer of investment (capital) good (which, recall, is freely mobile across sectors) are given by:

$$q_{i,t} = \xi_t \tag{28}$$

$$p_{i,t} \, x_{i,t} = \xi_t d_i \prod_{j=1}^{J} x_{j,t}^{d_j} \tag{29}$$

$$\sum_{i=1}^{J} z_{i,t} = \prod_{j=1}^{J} x_{j,t}^{d_j} \tag{30}$$

where ξ_t is the Lagrange multiplier for the technology constraint in the investment good producer's problem. Note that perfect capital mobility across sectors implies that the price of the capital across sectors is equal (Equation 28).

C. Market Clearing Conditions

Market clearing in the sectoral good markets implies

$$y_{i,t} = c_{i,t} + x_{i,t} + g_{i,t} + \sum_{j=1}^{J} n_{j,i,t} \tag{31}$$

and for money it implies

$$m_t = 1 \tag{32}$$

Furthermore, supply and demand of sectoral inputs of labor and capital ($h_{i,t}$ and $k_{i,t}$ respectively) will be equal in equilibrium.

3.4. Stationary Equilibrium

A *stationary equilibrium* for this economy is defined as the price vectors $\left\{\mathbf{p}_t, \mathbf{r}_t\right\}_{\mathbf{t=0}}^{\infty}$, prices $\left\{q_t, w_t\right\}_{t=0}^{\infty}$, quantity vectors $\left\{\mathbf{c}_t, \mathbf{h}_t, \mathbf{k}_t, \mathbf{x}_t, \mathbf{z}_t, \mathbf{y}_t\right\}_{\mathbf{t=0}}^{\infty}$ and shadow prices $\{\lambda_t, \gamma_t, \xi_t\}_{t=0}^{\infty}$ such that for a given vector $\{\mathbf{A}_t\}_{t=0}^{\infty}$ of technology shifts, given money supply growth $\left\{\nu_t\right\}_{t=0}^{\infty}$, and given sectoral government (autonomous) expenditures denoted by a vector $\{\mathbf{g}_t\}_{t=0}^{\infty}$, the following conditions hold:

(1) the consumer's problem is solved; that is, conditions (19) - (24) are satisfied;

(2) the necessary and sufficient conditions for the firm's profit maximization, (25) - (27) are satisfied;

(3) the investment good producer maximizes her profits; that is, conditions (28) - (30) are satisfied;

(4) all markets clear.

3.5. Steady State and Near Steady State Dynamics

We solve for some key steady-state ratios that will be useful for calibrating the model. The share of real consumption expenditures of industry i's output is given by

$$\frac{c_i}{y_i} = \frac{\theta_i}{\phi_i} \tag{33}$$

where ϕ_i is the i^{th} element of the vector $\mathbf{\Phi} = (\mathbf{I} - \mathbf{d}\rho'\alpha - \mathbf{a}' - \mathbf{g})^{-1}\theta$, $\mathbf{d}$ is the vector of the sectors' shares of aggregate gross fixed investment expenditures, **g** is the vector of sectoral government expenditure shares in sectoral output (which is taken to be exogenous), ρ is a vector of ρ_is where $\rho_i = \sum_{j=1}^{J} \frac{\beta(\eta_k - 1 + \delta_j)}{\eta_k - (1-\delta_i)\beta}$, α is a J × J diagonal matrix of output elasticities of capital in J sectors, and θ is the vector of sectoral shares of aggregate nominal consumption expenditures. The term ϕ_i also represents sector i's share of total nominal expenditures. The fraction of sector i's output that is used as material input in industry j is given by:

$$\frac{n_{j,i}}{y_i} = a_{j,i} \frac{\phi_j}{\phi_i} \tag{34}$$

while the fraction of sector i's output that is used for investment is given by:

$$\frac{x_i}{y_i} = d_i \sum_{j=1}^{J} \rho_j \alpha_j \frac{\phi_j}{\phi_i} \tag{35}$$

In our analysis below, we analyze the local dynamics around the steady state when the economy faces alternative shocks, both sectoral and aggregate. Sectoral technology (represented by $A_{i,t}$s in the model), sectoral government expenditures (denoted by $g_{i,t}$s), and aggregate money growth (denoted by ν_t) are all modeled as first order autoregressive processes. Appendix 1 describes the linear system used to approximate the dynamics of the model.

4. Calibration

The Input-Output (I-O) Tables provide a well-defined structure at various levels of disaggregation that is representative of the way the sectors interact among themselves. In 1987 benchmark I-O tables, 95 industries are covered at the two-digit level, while details are provided for 480 industries at the six-digit level. Unfortunately, we do not have time series data on prices, output, and inputs for this classification of industries. However, as we use an extended version of KLEM data set to calculate sectoral Solow residuals (which represent productivity in the setup of our model), we consolidate the 1987 I-O table to thirty-five sectors to conform to the classification of sectors in the KLEM data set.

From the consolidated I-O table, we calibrate the matrix, **a**, which describes the input-output relationships between the sectors. The share of compensation of employees in (nominal) output is computed for each of the thirty five sectors from the 1987 I-O table, and used to calibrate the vector of output elasticities of labor, **b**. Then the vector of capital share coefficients, α, is recovered from the assumption of constant returns to scale, i.e. $\alpha_i = 1 - \sum_{j=1}^{J} a_{i,j} - b_i$. The vector **d**, which describes the sectors' shares of aggregate gross fixed investment expenditures, and the share of each sector's output (ϕ_i) in aggregate output are also calculated from the 1987 I-O table. The fraction of each sector's output purchased by government, g_i, is taken to be 1 minus the shares of consumption, investment, and material input usage of that sector's output. Average growth in the sectoral Solow residuals over our sample and labor shares are in turn used to calibrate the vector of labor-augmenting technological growth factor, η_μ. The sectoral depreciation rates, δ_i s, are taken from Horvath (2000). These, in turn, are used to calculate the vector of sectoral shares of aggregate consumption expenditures, θ. Finally, we set the discount factor, $\beta = 0.95$, which is comparable to those used in other studies.

In order to estimate a stochastic process for "sectoral government expenditures", we need time series data on sectoral government purchases. Unfortunately, annual time series for sectoral government expenditures alone is not available. Therefore, to obtain annual time series data on "sectoral government expenditures" (as opposed to steady state values) we take the residual of sectoral real GDP (value added) after subtracting off sectoral consumption and gross fixed investment. Since there is no way that we can decompose these residuals into government expenditures and net exports these data are subject to the volatility that is present in net exports. Thus, these might be more accurately described as "sectoral autonomous expenditure". A detailed discussion on our methodology to construct this variable is presented in Appendix 2. Data on aggregate money base for the period from 1947 to 1989 are obtained from the DRI-Pro database. We set $\nu = \exp(0.0486)$, which implies an approximate growth rate of 5 percent for money supply.

In order to calibrate shocks to sectoral technology and sectoral autonomous expenditures, we first detrend these variables by subtracting the respective trend components obtained from linear regressions of these variables on a constant and time. We then estimate AR(1) models for the detrended variables and save the residuals from these regressions. For money growth rates, we simply estimate an AR(1) model for the growth rates of money stock and save the residuals. In most of the experiments below, shocks will be set to resampled residuals. By using the empirical distribution of shocks, we can maintain in our simulations the covariance structure of shocks seen in the data, without having to specify a particular joint distribution for these shocks.

Table 2 lists the thirty-five sectors and presents the fractions of each sector's output that goes for intermediate uses or for various final uses such as consumption, investment, government purchases and net exports in 1987. As we can see from column 1 of the table, for twenty-one sectors more than half of their output is used as material inputs in other sectors. Among them, 'metallic ores mining', 'crude petroleum and natural gas', 'nonmetallic minerals mining', 'stone' and 'primary metal' sectors supply almost all of their output for intermediate uses. For ten sectors, more than half of their output is used for consumption. 'Food and kindred products', 'tobacco products', 'apparel', 'footwear and leather products' and 'miscellaneous manufacturing products' are predominantly consumption good producing sectors. On the other hand, 'construction' and 'machinery manufacturing' are predominantly investment good producing sectors. Note that the government purchases nearly half of the total output of the 'transportation equipment' sector. In the 'footwear and leather products' sector, more than its domestic output is imported into the U.S. Furthermore, more than 40 percent of the domestic production of 'crude petroleum and natural gas' and 'miscellaneous manufacturing products' is imported. Since crude oil is mainly used as intermediate inputs, one would expect that external disturbances in this sector would have significant impact on the U.S. domestic production of petroleum products. As we can see, none of these sectors is a major net exporter in terms of the fraction of its output that is being exported.

5. Simulation Results

5.1. Average Sectoral Correlations between Price, Output and Solow Residual

Table 3 displays average price-output growth correlations across the thirty-five sectors for four separate experiments. For each of these experiments, we simulate the model 1,000 times for 40 periods (the length of our sample period), with an initial startup of 50 periods to eliminate any potential effect of initial conditions. For each simulation, we calculate average correlations along with the interior 90 percent interval of the distribution of simulated correlations. Table 3 also includes the average sectoral price-Solow residual correlation and the average output-Solow residual correlation.

Examining column (2) in Table 3, we observe that the model with the full complement of sectoral technology, sectoral demand, and money growth shocks yields average (across sectors) sectoral price-output and sectoral price-Solow residual correlations that are quite close to those found in the data. For both these correlations, the interior 90 percent of the simulated correlations include the actual sectoral price-output and price-Solow residual correlations. On the other hand, the average sectoral output-Solow residual correlation generated by the model is too high relative to that in the data.

To see how the different features of the model affect the average price-output correlation, we consider three additional experiments. First, we examine a model in which there is no sectoral input-output interaction, sectoral technology shocks are independent of each other, and there are no money growth or sectoral demand shocks. Of the specifications of the model we examine, this is the closest to the simple partial equilibrium example discussed in section 2. As expected the average sectoral price-output, sectoral price-Solow residual correlations are close to –1 (see column 3 of Table 3). That the average correlation is not equal to minus one is due to the fact that output of some sectors is used as capital goods. Positive technology shocks, in any sector, tend to increase the demand for capital. This, in turn, acts to increase demand in sectors that produce capital goods.

As we add sectoral input-output interaction (column 4), we see that sectoral price-output and price-Solow residual correlations drop (in absolute value).[4] In this case, a sectoral technology shock will also affect the demand for goods that are used as intermediate inputs. Adding sectoral demand shocks (column 5), introduces shocks that have direct effects on sectoral demands lowering price-output and price-Solow residual correlations further. For this experiment, we obtain simulated price-output correlations that are getting close to that observed in the data. Finally, by comparing column (5) with column (2), we see the incremental effect of money growth shocks. Not surprisingly, money growth shocks tend to lower average sectoral price-output and price-Solow residual correlations. In sum, no single feature is responsible for generating sectoral price-output correlation similar to that in the data: sectoral input-output relationships, sectoral demand shocks, and money growth shocks all have implications for sectoral price-output correlations.

How well does our sectoral model replicate some of the standard aggregate business cycle statistics? Table 4 presents variances, relative variances, cross-correlations with aggregate value-added (GDP). Even though the focus of our analysis is not on these aggregate real relationships, they provide a benchmark with which we can

[4] We also allow sectoral technology shocks to be correlated. This is accomplished by resampling the vector of residuals from a AR(1) fitted to actual Solow Residuals.

compare the aggregate fluctuations implied by our model with those in the business cycle literature.[5] While our model does well for many of these aggregate statistics, it fails dramatically in two crucial respects - the relative volatility of investment is too high and the correlation between output and consumption is too low. This appears to be true regardless of whether we include sectoral interactions, sectoral demand shocks, or money growth shocks.

5.2. Individual Sectoral Price-Output Correlations

To get a better sense of what is happening at the sectoral level, we present in Table 5 price-output correlations for each sector individually. Comparing the price-output correlation from the data (column 1) with that implied by the baseline model (column 2), we observe that, even though the averages (across sectors) are similar, there are several sectors in which the data and model are dramatically different. These instances most often occur in sectors that are important investment good producers - construction, furniture, machinery, electric machinery, and instruments - or are important intermediate inputs to these sectors - stone, primary metals, and fabricated metals. Here the model generally implies a positive price-output correlation while the actual correlations are generally negative.

For these sectors, as we suggested above, any technology shock that increases demand for capital will increase demand in these capital good producing sectors. This is clear from the model in which there are independent technology shocks, no input-output structure, and no demand shocks (column 3). With log utility, no input-output structure, and no autonomous expenditures, nominal expenditures ($p_{i,t}y_{i,t}$) in sectors whose output is

[5] As we examine annual data, we linearly detrend the national income and product account data in order to isolate the cyclical behavior rather than employ Hodrick-Prescott filter.

not used as an investment good is proportional to the money stock. Thus, unless a sector's output is used for investment (see Table 2), price-output correlations are minus one. As we add sectoral input-output interaction (column 4), price-output correlations for all sectors fall. In fact, as we add sectoral input-output relationships and sectoral demand shocks (column 5) to the model, the price-output correlations for many of the investment good sectors and sectors that produce important material inputs to investment goods become positive.

That our model generates positive price-output correlations for investment goods and important investment good inputs may be related to the very high investment volatility implied by the model. In the baseline model, while the current (time t) sectoral capital stock is fixed, it is costless to move capital in and out of a sector at time t+1. Because of the sectoral capital mobility, the model implies counterfactually large changes in investment demand. This leads to high aggregate investment volatility, as well as large changes in the demand for capital good producing sectors and in the demand for inputs of those sectors.

6. Responses of Sectoral Output and Prices to Aggregate and Sectoral Shocks

In this section we try to understand more deeply how different sectors respond to both aggregate and sectoral shocks. In particular, we are interested in knowing to what extent a sector's role as an input or final good producer determines how it responds to alternative types of shocks, and how shocks in that sector are transmitted to other sectors.

Figure 2 illustrates the distribution of responses of output and price across sectors to a money supply growth shock at various time horizons: 0, 1, 5 and 10. From Figure 2, we observe that prices increase in all sectors at all time horizons. Monetary shocks while generally increasing all prices do, however, have consequences for current relative price changes. At shorter horizon, sectors producing goods for investment or intermediate uses experience relative price increases. At longer horizons, price changes are about one percent – approximately equal to the magnitude of the change in the money stock – in almost all sectors. On the other hand, money growth shocks result in output increases in some sectors and declines in others. Note that the sectors experiencing a rise in output at shorter horizons (i.e. at period 0 and 1) produce goods that are used either for investment or for intermediate uses while the sectors producing consumption goods experience some of the highest output declines. It appears that at short time horizon, a money growth shock causes household to substitute away from consumption goods and into investment goods. Because money growth shocks are persistent (but not permanent), anticipation of inflation will lead people to substitute away from consumption goods that require cash. The net result is that demand for investment goods and for some material inputs rise.

Figures 3 and 4 plot the cross-section responses of output and prices to a technology shock to material input producing sectors. In particular, we are considering shocks to the crude oil and natural gas sector, and the rubber sector. Note that almost the entire output of the crude oil sector and 90 percent of the total output of the rubber sector is used as material inputs (see column 1 of Table 2). A positive technology shock to the material input producing sector looks very much like a typical supply side shock in that sector - price declines and output rises. Furthermore, the effects in other sectors look very much like supply shocks. For crude oil, two other sectors, namely petroleum refining and gas utilities which receive the bulk of their material inputs from the crude oil sector, experience large changes in their output and prices. The effects in other sectors are negligible. We see similar pattern of output and price responses at longer horizons as well; however, the magnitudes of output and price responses decrease over time. The effects on prices and output in other sectors of a shock to the Rubber sector also look like those of a traditional supply shock.

Figures 5 and 6 illustrate the responses of sectoral output and prices to a technology shock to consumption good producing sectors. We chose the motor vehicle sector and the wholesale and retail trade sector as the former sells about 52 percent of its output and the latter sells about 62 percent of its output for consumption (see column 2 of Table 2). Figures 5 and 6 illustrate the results of this experiment. As evident from these figures, a technology shock in a consumption good sector acts as a supply shock in that sector: output increases and price decreases at all horizons. On the other hand, the responses of price and output in other sectors at short horizons are suggestive of demand shocks; those sectors' prices and outputs rise. Immediately after a consumption good sector experiences a positive technology, demand for other goods that are either intermediate inputs or investment goods appears to increase. However, over time these temporary demand disturbances disappear. At longer horizons, the responses in these sectors are more like supply shocks though the magnitudes of the responses of output and prices are relatively small.

Figures 7 and 8 present the responses of sectoral output and prices to a technology shock to investment good producing sectors. We choose construction and machinery sectors for this experiment. As we see from column 3 of Table 2, 61 percent of the output of the construction sector and 51 percent of the machinery sector output is used for investment. As a result of a positive technology shock, while output rises and price declines in the construction sector (and in the machinery sector) most of the other sectors experience output increases and price increases at short time horizons. Thus the effects of a technology shock in an investment good producing sector on other sectors' outputs and prices at short horizons are more reminiscent of demand shocks. Again, a positive technology shock to an investment good producing sector increases demand for the goods mainly used as material inputs or investment goods in that sector. However, as the prices in the investment good producing sectors continue to decrease, the production costs in other sectors decrease, and consequently at longer horizons (time horizon 10) the output and price responses are suggestive more of supply shocks.

That sectoral technology shocks can look like demand shocks in other sectors may hold at the aggregate level as well. Figure 9 presents the response of aggregate value added (GDP) and the valued added deflator to technology shocks in the rubber sector and to machinery sectors. GDP rises and the deflator falls in response to a technology in intermediate good producing sector (rubber). On the other hand, the deflator may rise at least initially in response to a shock to a capital good producing sector (machinery). One implication is that if aggregate technology shocks are akin to a linear combination of sectoral technology shocks, then positive technology shocks may not have large initial aggregate price effects as the positive and negative sectoral price effects tend to cancel each other out. Thus, even though prices are perfectly flexible, the aggregate price level may appear to respond sluggishly to a technology shock.

7. Summary and Conclusions

In this paper, we examined the relationship between sectoral output growth and price changes. We find that price changes and output growth, both at the aggregate and the sectoral level, are on average negatively correlated. We also find that a flexible price, multisector, dynamic general equilibrium model is capable of reproducing the average sectoral price-output correlation. Both sectoral technology and sectoral autonomous expenditure shocks play important roles in generating the negative price-output correlation. We show that technology shocks to sectors producing goods that are used primarily as inputs to other sectors look like traditional supply shocks, prices fall and outputs rise. On the other hand, technology shocks to investment or consumption goods sectors can, at least in the short-term horizons, look like demand shocks in other sectors, prices and outputs rise while at longer horizons they take the appearance of supply shocks.

Appendix 1

Linear Approximation and State Space Solution of the Multisector Flexible Price Model

In this appendix we show how we approximate the optimizing conditions (19) - (32) linearly around a steady state and solve the resulting linear dynamic system. Note that the percentage deviations of the variables from steady state levels are denoted by a circumflex (^). Combining equations (19), (22), (24) and (32) and taking linear approximation we obtain,

$$\hat{c}_{i,t} + \hat{p}_{i,t} = 0 \tag{A.1}$$

We substitute for $y_{i,t}$ from the production function into (25) and solve for w_t. Then substituting for w_t into (20) and linearizing, we obtain

$$(1-b_i)\hat{h}_{i,t} + \sum_{i=1}^{J} sh_i \hat{h}_{i,t} - \sum_{j=1}^{J} a_{i,j} \hat{n}_{i,j,t} - \hat{p}_{i,t} = \hat{A}_{i,t} + \alpha_i \hat{k}_{i,t} - \underset{t}{E} \hat{v}_{t+1} \tag{A.2}$$

where sh_i is the ratio of labor input in sector i to leisure. Substitution for $y_{i,t}$ from the production function into (28) and linear approximation yields

$$b_i \hat{h}_{i,t} + \sum_{j=1}^{J} a_{i,j} \hat{n}_{i,j,t} - \hat{n}_{i,j,t} + \hat{p}_{i,t} - \hat{p}_{j,t} = -\alpha_i \hat{k}_{i,t} - \hat{A}_{i,t} \tag{A.3}$$

Substituting for ξ_t into (29) from (28) and then linearizing we obtain,

$$\hat{x}_{i,t} - \sum_{j=1}^{J} d_j \hat{x}_{j,t} + \hat{p}_{i,t} = \hat{q}_{i,t} \tag{A.4}$$

Linear approximation of the market clearing condition (31) yields

$$- sc_i\ \hat{c}_{i,t} + b_i\ \hat{h}_{i,t} + \sum_{j=1}^{J} a_{i,j}\ \hat{n}_{i,j,t} - \sum_{j=1}^{J} sn_{j,i}\ \hat{n}_{j,i,t} - sx_i\ \hat{x}_{i,t} = -\alpha_i\ \hat{k}_{i,t} - \hat{A}_{i,t} + (1 + \tilde{g}_i)\frac{g}{y_i}\hat{g}_t + \frac{g}{y_i}\hat{g}_{i,t} \quad (A.5)$$

where sc_i is the fraction of sector i's output allocated to consumption. Similarly, $sn_{j,i}$ is the fraction of good i output used as material inputs in sector j, and sx_i is its fraction used for investment. Note that in order to linearize the market clearing condition (31), we first re-write the equation as follows:

$$y_{i,t} = c_{i,t} + x_{i,t} + g_t \times \left(\frac{g_{i,t} - g_t}{g_t}\right) + g_t + \sum_{j=1}^{J} n_{j,i,t} \quad (A.6)$$

We define $\tilde{g}_{i,t} = \dfrac{g_{i,t} - g_t}{g_t}$, where $g_{i,t}$ is the government (autonomous) expenditure on good i in period t and g_t is the aggregate government expenditures. Therefore, $\tilde{g}_{i,t}$ can be interpreted as percentage deviation of sectoral government expenditures from the mean. We substitute for $y_{i,t}$ from the production function into (26) and solve for $r_{i,t}$. Then substituting for $r_{i,t}$ into (21) and linearizing, we obtain

$$E_t\left[\begin{array}{l}\left(1-(1-\delta_i)\frac{\beta}{\eta_k}\right)(\alpha_i - 1)\hat{k}_{i,t+1} + \left((1-\delta_i)\frac{\beta}{\eta_k}\right)\hat{q}_t + \left(1-(1-\delta_i)\frac{\beta}{\eta_k}\right) \\ \times\left\{b_i\ \hat{h}_{i,t+1} + \sum_{j=1}^{J} a_{i,j}\ \hat{n}_{i,j,t+1} + \hat{p}_{i,t+1} + \hat{A}_{i,t+1}\right\} - \hat{q}_t - \hat{v}_{t+2} + \hat{v}_{t+1}\end{array}\right] = 0 \quad (A.7)$$

We substitute for $z_{i,t}$ in (30) from the capital accumulation technology and linearize around steady state to obtain

$$\sum_{i=1}^{J} sz_i \frac{\eta_k}{\{\eta_k - (1-\delta_i)\}}\hat{k}_{i,t+1} = \sum_{i=1}^{J} \frac{sz_i(1-\delta_i)}{\{\eta_k - (1-\delta_i)\}}\hat{k}_{i,t} + \sum_{i=1}^{J} d_i\ \hat{x}_{i,t} \quad (A.8)$$

where sz_i is the share of investment in sector i to aggregate investments in the economy.

Using notation in King, Plosser, and Rebelo (1987), equations (A.1) - (A.5) can be written as:

$$\mathbf{Mcc}\ \mathbf{cl_t} = \mathbf{Mcs}\ \mathbf{s_t} + \mathbf{Mce}\ \mathbf{e_t} \tag{A.9}$$

where **Mcc** is a $(J^2+4J) \times (J^2+4J)$ matrix of the coefficients; $\mathbf{cl_t}$ is a $(J^2+4J) \times 1$ vector of control variables, namely c_i, h_i, x_i, $n_{i,j}$ and p_i , and i,j = 1,2,3,….J; **Mcs** is a $(J^2+4J) \times (J+1)$ matrix of coefficients; $\mathbf{s_t}$ is a $(J+1)\times 1$ vector of J predetermined variables, k_i, i= 1,2,….J, and one non-predetermined variable, q; **Mce** is a $(J^2+4J) \times (2J+1)$ matrix of coefficients and $\mathbf{e_t}$ is a $(2J+1) \times 1$ vector of exogenous variables. Given this system of equations, (A.9) can be solved for the control variables as functions of predetermined, non-predetermined and exogenous variables. Given these optimal solutions, equations: (A.7) and (A.8) imply a first-order dynamic system in $\mathbf{s_t}$, i.e. in k_is and q,

$$\underset{t}{\mathrm{E}}\,\mathbf{s}_{t+1} = \mathbf{W}\mathbf{s}_t + \mathbf{R}\,\underset{t}{\mathrm{E}}\,\mathbf{e}_{t+1} + \mathbf{Q}\mathbf{e}_t \tag{A.10}$$

where **W** is a $(J+1) \times (J+1)$ matrix and **R** and **Q** are $(J+1) \times (2J+1)$ matrices. To obtain the solution to this system of difference equations we decompose **W** as $\mathbf{P}\mu\mathbf{P}^{-1}$, where **P** is the matrix of eigenvectors of **W** and μ is a diagonal matrix with the eigenvalues on the diagonal. For our system since there is only one non-predetermined variable (q_t), in order for this system to have a unique solution there should be just one eigenvalue of **W** outside the unit circle.

After algebraic manipulation, the optimal time path for capital accumulation may be written as follows:

$$\hat{k}_{t+1} = \pi_{kk}\,\hat{k}_t + \pi_{ke}\,\hat{e}_t \tag{A.12}$$

where π_{kk} and π_{ke} are respectively J × J and J × (2J+1) matrices of coefficients. These coefficients are complicated functions of the underlying parameters of preferences and technology. The dynamics of capital accumulation depend on the previous period's capital stock, current exogenous factors and on the entire future time path of these factors.

Appendix 2

Construction of Data on Sectoral Autonomous Expenditures

For the simulation of the model, we need to calibrate shocks to the sectoral government expenditures, which are assumed to be exogenous. Data on government expenditures by sectors are, however, not readily available. Therefore, we decompose sectoral real GDP into three final expenditure components, namely, consumption, investment and government expenditures. This decomposition scheme exactly matches the market clearing conditions of our model. The method we use for decomposition involves the following steps:

1) We obtain data on 'personal consumption expenditures' and 'gross fixed investments' by major type of products from unpublished tables of the National Income and Product Accounts (NIPA) for a period from 1947 to 1997. Note that consumption data are available for 83 product categories and investment data are available for 26 product categories.
2) These consumption and investments by products are then mapped into consumption and investments by two-digit input-output (I-O) industries using a mapping scheme

outlined in Survey of Current Business (April, 1994) for 1987 Benchmark Input-Output Tables. Note that there are ninety-five two-digit I-O industries.

3) These consumption and investments data are then consolidated to match thirty-five sectors of the KLEM data set.

4) We obtain data on GDP by 2-digit SIC industries from the National Income and Product Accounts (NIPA) "Income, Employment and Product by Industry" unpublished detail tables and consolidate, wherever necessary, for thirty-five sectors.

5) Data on government expenditures by sectors are then obtained as residuals by subtracting consumption and investments from sectoral GDP. Note that all these values are in current dollars. In order to convert them into 1987 constant dollar we deflate the government expenditures by the price indices obtained from the KLEM data set.

Note that sectoral GDP also includes a 'net exports' component and it is impossible to separate out this component from the residuals we obtain in the final step. Therefore, we prefer the nomenclature 'sectoral autonomous expenditures' to 'sectoral government expenditures'. Sectors that have relatively higher shares of net exports in their respective total GDP will be subject to volatility that arises from changes in international market conditions.

Table 1. Summary Statistics of Correlations across Sectors

	Mean (1)	Median (2)	Standard Deviation (3)	Minimum (4)	Maximum (5)
Corr(Δp_i ,Δy_i)	-0.27 (0.040)	-0.31	0.24	-0.60	0.33
Corr(Δp_i ,ΔA_i)	-0.51 (0.033)	-0.53	0.19	-0.89	0.01
Corr(Δy_i ,ΔA_i)	0.45 (0.045)	0.49	0.27	-0.18	0.91

Note: Sample period 1949-1989. Estimated standard errors are in parentheses.

Table 2. Intermediate and Final Uses of Sectoral Output in 1987

Sector	Fractions of output in different uses				
	Intermed (1)	Consum (2)	Invest (3)	Govt.pur (4)	Net exp (5)
1 Agriculture	0.87	0.12	0.00	0.01	0.04
2 Metallic ores mining	1.07	0.00	0.07	-0.02	-0.12
3 Coal mining	0.84	0.01	0.00	0.00	0.10
4 Crude oil and natural gas	1.43	0.00	0.00	0.00	-0.40
5 Nonmetallic minerals mining	1.01	0.00	0.00	0.00	-0.01
6 Construction	0.20	0.00	0.61	0.19	0.00
7 Food and kindred products	0.38	0.61	0.00	0.02	-0.02
8 Tobacco products	0.14	0.79	0.00	0.00	0.06
9 Textile	0.85	0.12	0.05	0.01	-0.05
10 Apparel	0.29	0.98	0.00	0.02	-0.31
11 Lumber and wood products	0.94	0.02	0.05	0.00	-0.04
12 Furniture and fixtures	0.11	0.53	0.42	0.05	-0.13
13 Paper	0.89	0.11	0.00	0.03	-0.04
14 Printing	0.63	0.27	0.00	0.08	0.00
15 Chemicals	0.66	0.28	0.00	0.05	-0.01
16 Petroleum refining and related prod	0.51	0.44	0.00	0.08	-0.05
17 Rubber	0.90	0.09	0.00	0.02	-0.02
18 Footwear, leather, and leather	0.41	1.55	0.00	0.02	-1.03
19 Stone	0.98	0.07	0.00	0.01	-0.08
20 Primary metal	1.08	0.00	0.00	0.01	-0.11
21 Fabricated metal	0.93	0.04	0.04	0.02	-0.03
22 Machinery	0.43	0.03	0.51	0.08	-0.05
23 Electrical machinery	0.61	0.23	0.20	0.09	-0.15
24 Motor vehicle	0.31	0.52	0.34	0.04	-0.27
25 Transportation equipment	0.19	0.09	0.09	0.48	0.12
26 Instruments	0.24	0.09	0.38	0.31	-0.03
27 Miscellaneous manufacturing	0.30	0.87	0.12	0.05	-0.42
28 Transport	0.59	0.25	0.01	0.06	0.09
29 Communications	0.51	0.39	0.03	0.06	0.02
30 Electric utility service	0.51	0.40	0.00	0.09	-0.01
31 Gas utility service	0.68	0.31	0.00	0.03	-0.02
32 Wholesale and retail trade	0.28	0.62	0.05	0.01	0.04
33 FIRE	0.38	0.57	0.02	0.02	0.02
34 Services	0.44	0.54	0.01	0.01	0.00
35 Government enterprises	0.56	0.39	0.00	0.04	0.00

Note: These ratios have been calculated from BEA's 1987 Benchmark I-O Use Table

Table 3. Simulation Results: Average Sectoral Correlations

	Data	Baseline Model	No I-O structure and independent technology shocks	I-O structure and correlated technology shocks	Add sectoral demand shocks
	(1)	(2)	(3)	(4)	(5)
$Corr(\Delta p_i, \Delta y_i)$	-0.27	-0.27 (-0.41, -0.14)	-0.84 (-0.87, -0.81)	-0.61 (-0.67, -0.55)	-0.39 (-0.49, -0.29)
$Corr(\Delta p_i, \Delta A_i)$	-0.51	-0.56 (-0.67, -0.45)	-0.92 (-0.95, -0.89)	-0.79 (-0.84, -0.74)	-0.68 (-0.75, -0.61)
$Corr(\Delta y_i, \Delta A_i)$	0.45	0.71 (0.67, 0.75)	0.84 (0.81, 0.87)	0.83 (0.81, 0.84)	0.75 (0.71, 0.78)

Note: The five and ninety-five percentiles for the simulations are in parentheses.

Table 4. Simulation Results: Aggregate Volatility and Aggregate Correlations

	Data (1)	Baseline Model (2)	No I-O structure and independent technology shocks (3)	I-O structure and correlated technology shocks (4)	Add sectoral demand shocks (5)
SD (Y)	0.05	0.06	0.03	0.06	0.06
SD(C) / SD(Y)	0.56	0.61	0.65	0.44	0.55
SD(X) / SD(Y)	1.48	5.83	5.70	5.16	5.70
SD(H) / SD(Y)	0.52	0.80	0.82	0.70	0.74
Corr (Y, C)	0.79	0.15	0.15	0.30	0.21
Corr (Y, X)	0.85	0.86	0.88	0.94	0.86
Corr (Y, H)	0.76	0.80	0.80	0.89	0.85

Note: The statistics for the U.S. economy in column 1 are calculated by aggregating the data for 35 sectors from the KLEM data set. Sample period 1947 - 1989.

Table 5. Simulation Results: Sectoral Price-Output Correlation

	Data (1)	Baseline Model (2)	No I-O Structure & independent technology shocks (3)	I-O structure & correlated technology shocks (4)	Add sectoral demand shocks (5)
1 Agriculture	-0.24	-0.86	-1.00	-0.99	-0.91
2 Metallic ores mining	0.28	-0.17	-0.99	-0.54	-0.26
3 Coal mining	0.07	-0.85	-1.00	-0.96	-0.92
4 Crude oil and natural gas	-0.17	-0.87	-1.00	-0.97	-0.91
5 Nonmetallic minerals	-0.52	-0.37	-1.00	-0.65	-0.58
6 Construction	-0.39	0.48	-0.29	0.53	0.54
7 Food and kindred produ	-0.56	0.09	-1.00	-0.98	0.08
8 Tobacco products	-0.20	-0.90	-1.00	-1.00	-1.00
9 Textile	0.10	-0.36	-0.96	-0.71	-0.51
10 Apparel	-0.44	-0.39	-1.00	-0.94	-0.59
11 Lumber and wood produ	0.33	0.17	-0.96	0.04	0.06
12 Furniture and fixtures	-0.38	0.50	-0.18	0.25	0.51
13 Paper	-0.18	-0.57	-1.00	-0.90	-0.80
14 Printing	-0.45	-0.60	-1.00	-0.97	-0.82
15 Chemicals	-0.42	-0.75	-1.00	-0.95	-0.89
16 Petroleum refining and	-0.18	-0.87	-1.00	-0.98	-0.93
17 Rubber	-0.26	-0.24	-1.00	-0.63	-0.45
18 Footwear, leather and	-0.23	-0.89	-1.00	-1.00	-0.98
19 Stone	-0.34	0.20	-1.00	-0.10	0.09
20 Primary metal	0.12	0.21	-1.00	-0.29	0.11
21 Fabricated metal	-0.27	0.32	-0.96	-0.09	0.22
22 Machinery	-0.18	0.52	-0.27	0.05	0.44
23 Electrical machinery	-0.32	0.49	-0.29	0.19	0.46
24 Motor vehicle	-0.31	-0.37	-0.64	-0.87	-0.55
25 Transportation equip	-0.05	-0.10	-0.02	-0.56	-0.30
26 Instruments	-0.17	0.51	-0.10	0.15	0.43
27 Misc. manufacturing	-0.51	-0.44	-0.88	-0.81	-0.65
28 Transportation	-0.32	-0.40	-1.00	-0.81	-0.52
29 Communications	-0.56	-0.65	-0.99	-0.90	-0.84
30 Electric utility service	-0.60	-0.30	-1.00	-0.67	-0.55
31 Gas utility service	-0.19	-0.77	-1.00	-0.92	-0.87
32 Wholesale and retail	-0.48	0.09	-0.95	-0.71	0.09
33 FIRE	-0.46	-0.20	-0.99	-0.86	-0.15
34 Services	-0.57	-0.40	-1.00	-0.77	-0.68
35 Govt. enterprises	-0.47	-0.75	-1.00	-0.99	-0.89
Mean	-0.27	-0.27	-0.84	-0.61	-0.39

Figure 1(a). Effects of a Positive Supply Shock

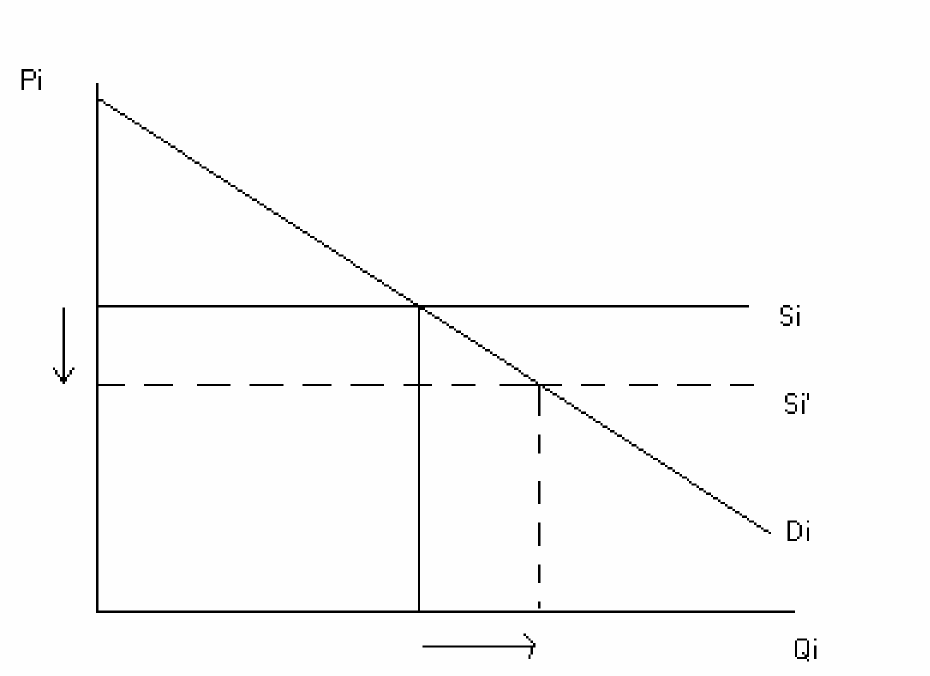


Figure 1 (b). Effects of a Positive Monetary Shock

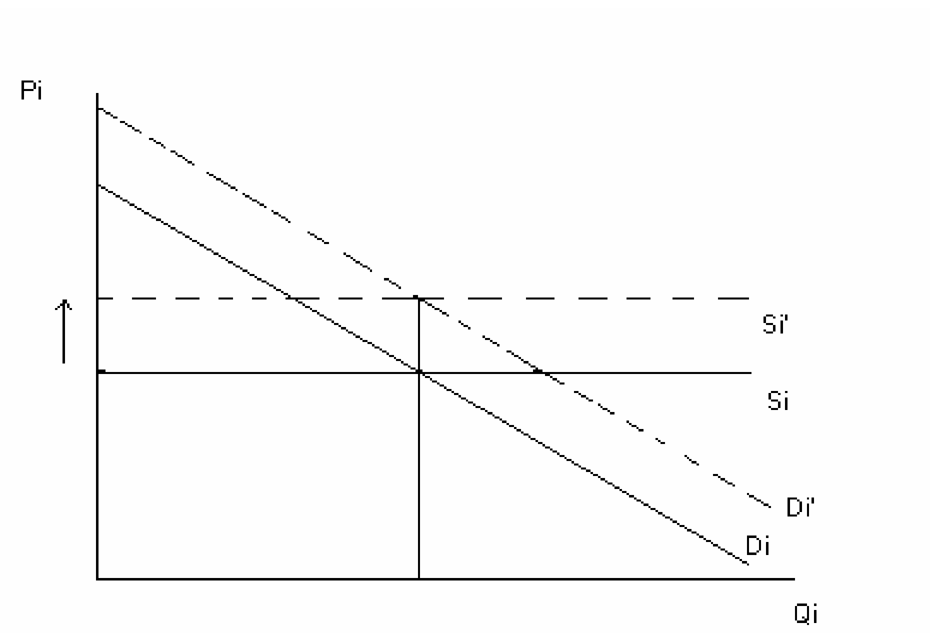
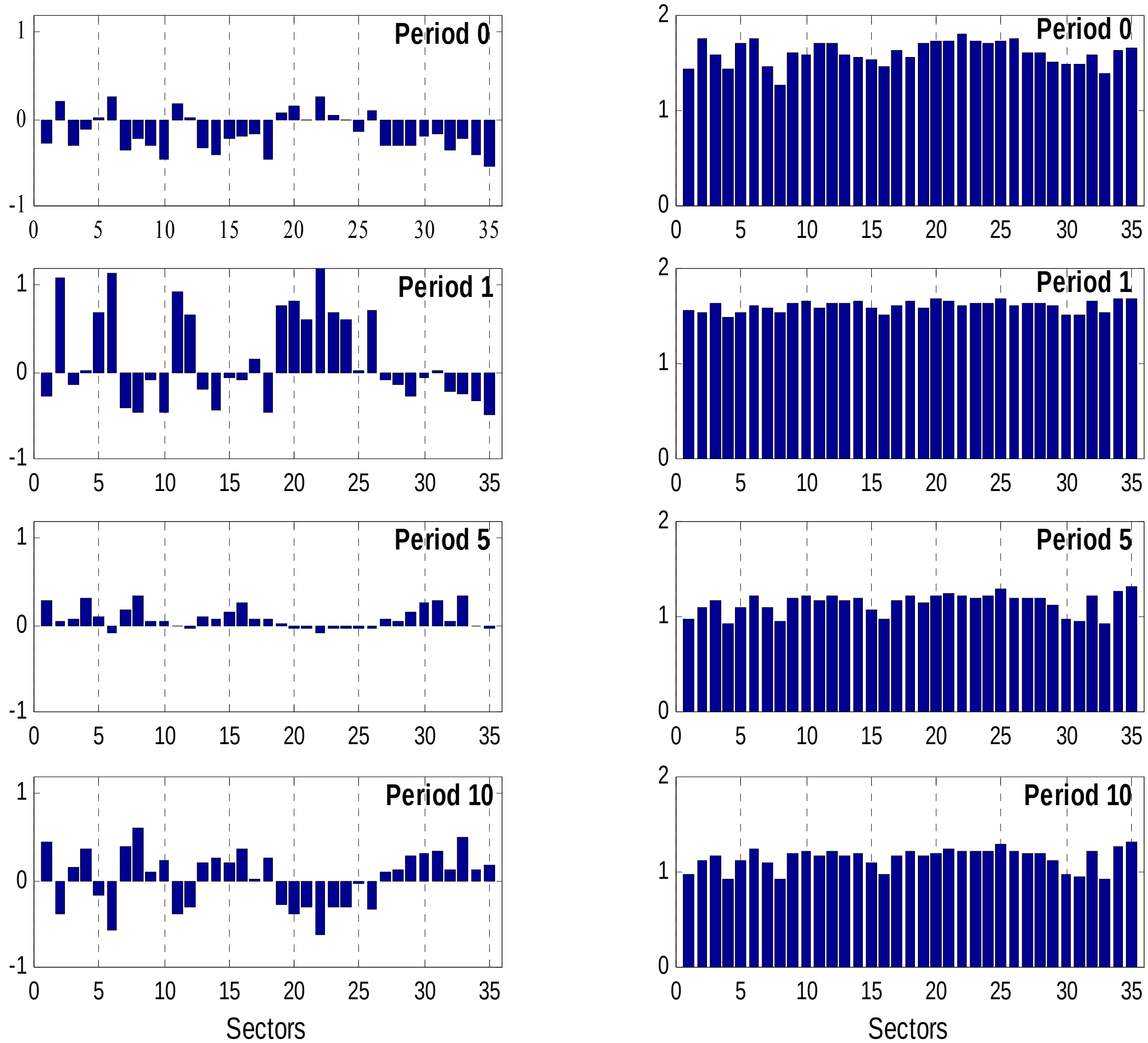


Figure 2. Sectoral Distribution of Impulse Responses to a Monetary Shock

Output Responses | Price Responses



Note: The height of each bar represents the percentage deviation of output (price) of the corresponding sector from its steady state level of output (price). Refer to Table 2 for a list of the 35 sectors represented along the horizontal axis.

Figure 3. Sectoral Distribution of Impulse Responses to a Technology Shock to the Crude Oil and Natural Gas Sector

Output Responses

Price Responses

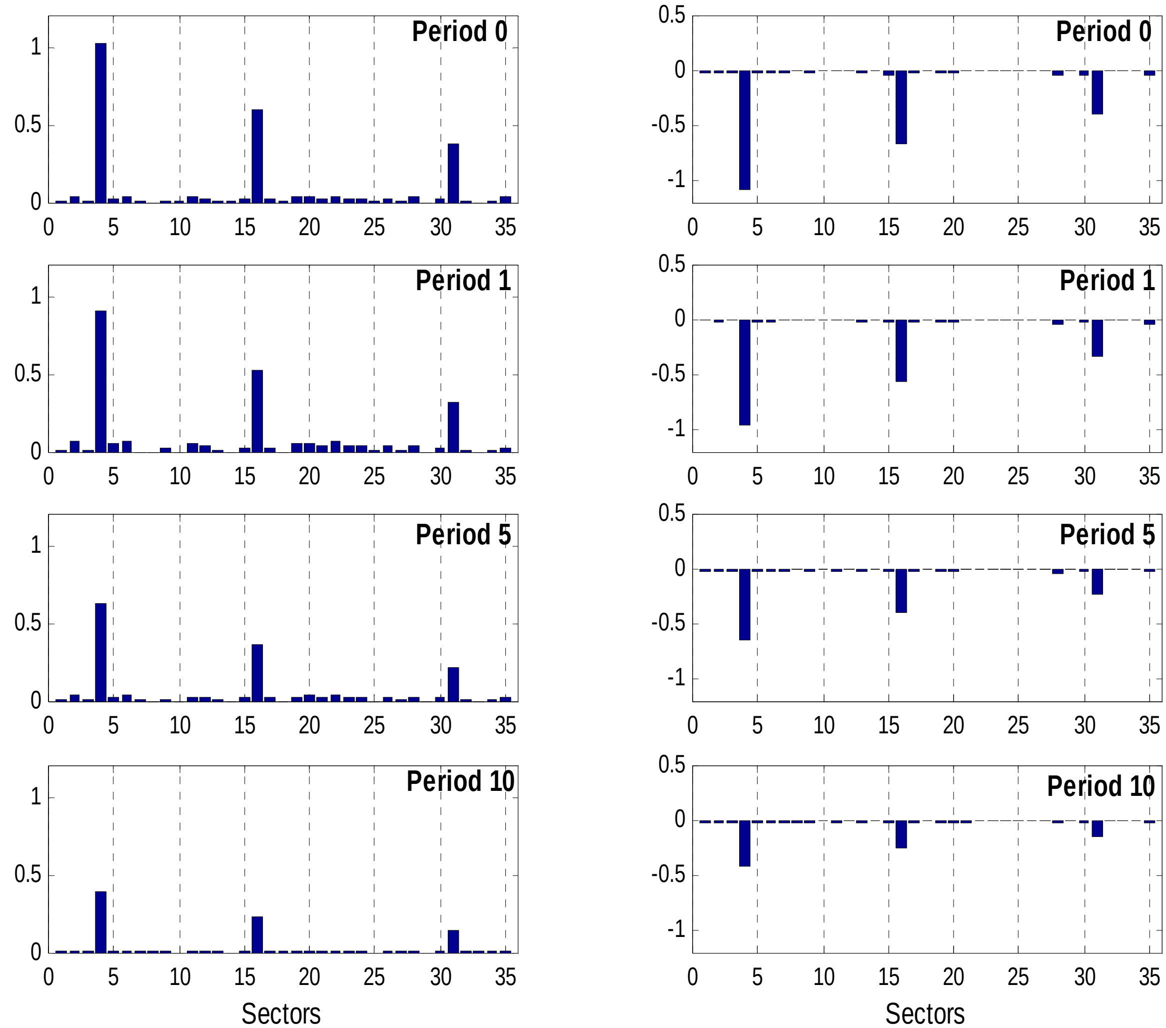


Figure 4. Sectoral Distribution of Impulse Responses to a Technology Shock to the Rubber Sector

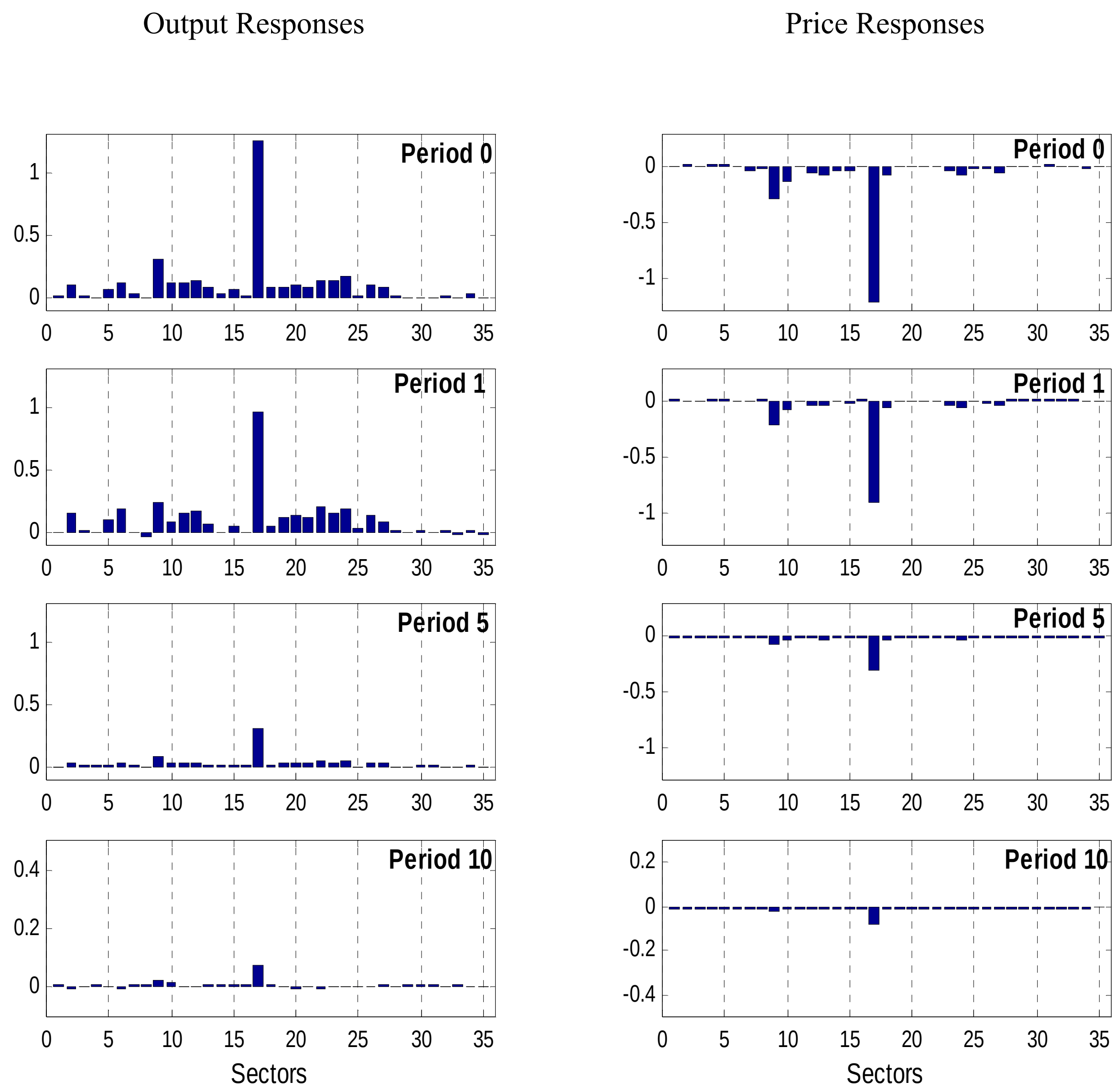


Figure 5. Sectoral Distribution of Impulse Responses to a Technology Shock to the Motor Vehicle Sector

Output Responses | Price Responses

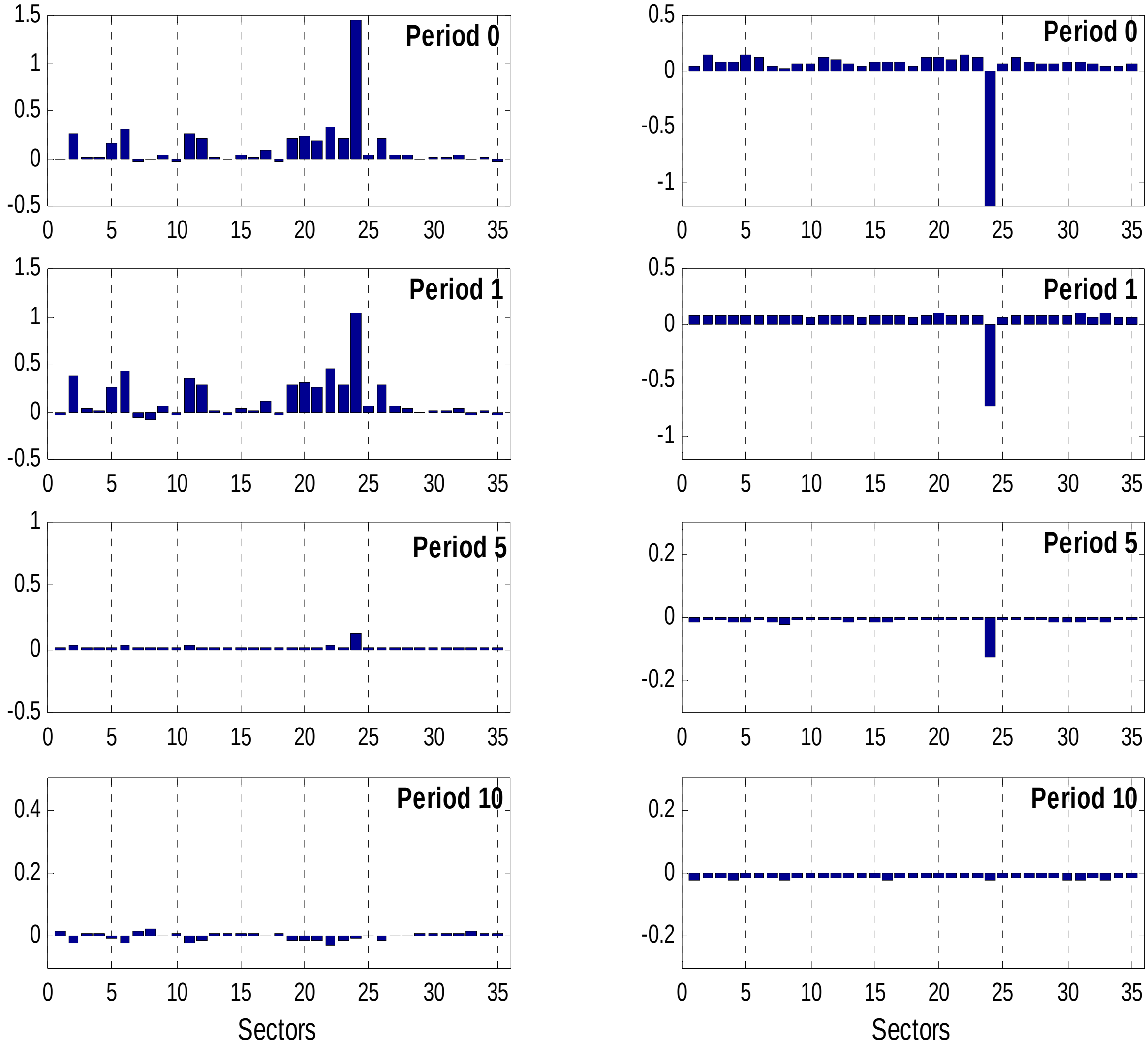


Figure 6. Sectoral Distribution of Impulse Responses to a Technology Shock to the Wholesale and Retail Sector

Output Responses

Price Responses

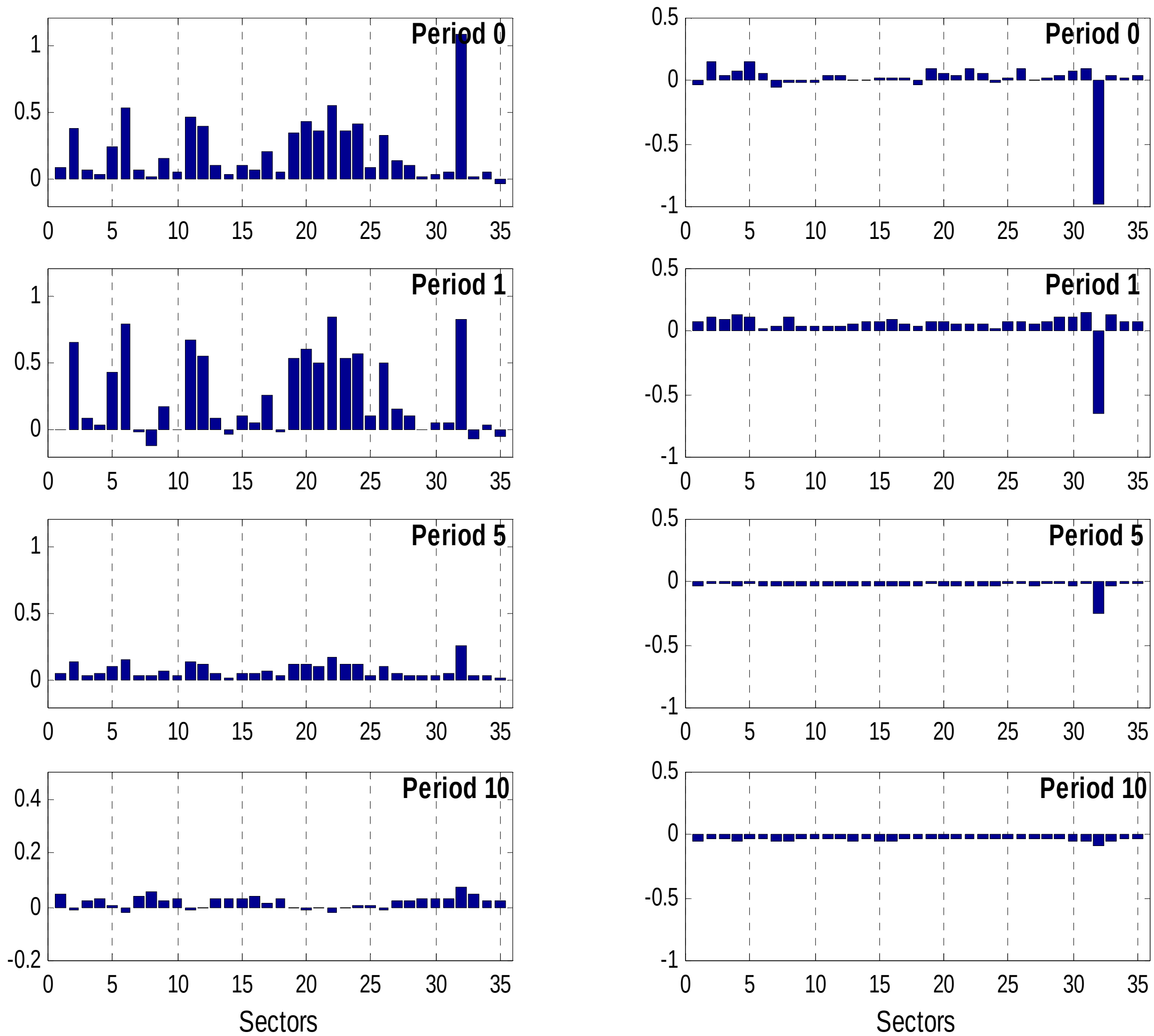


Figure 7. Sectoral Distribution of Impulse Responses to a Technology Shock to the Construction Sector

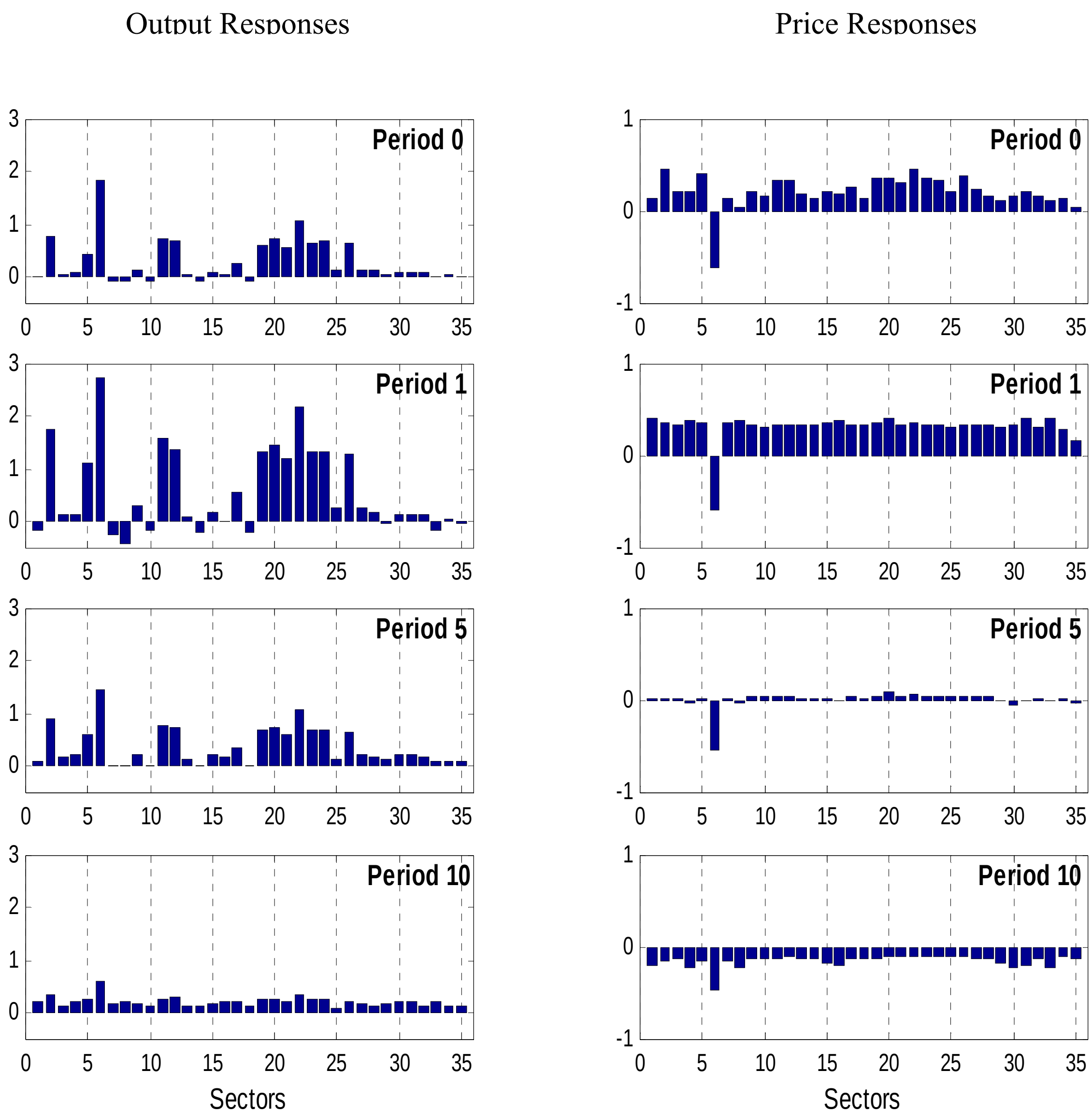


Figure 8. Sectoral Distribution of Impulse Responses to a Technology Shock to the Machinery Sector

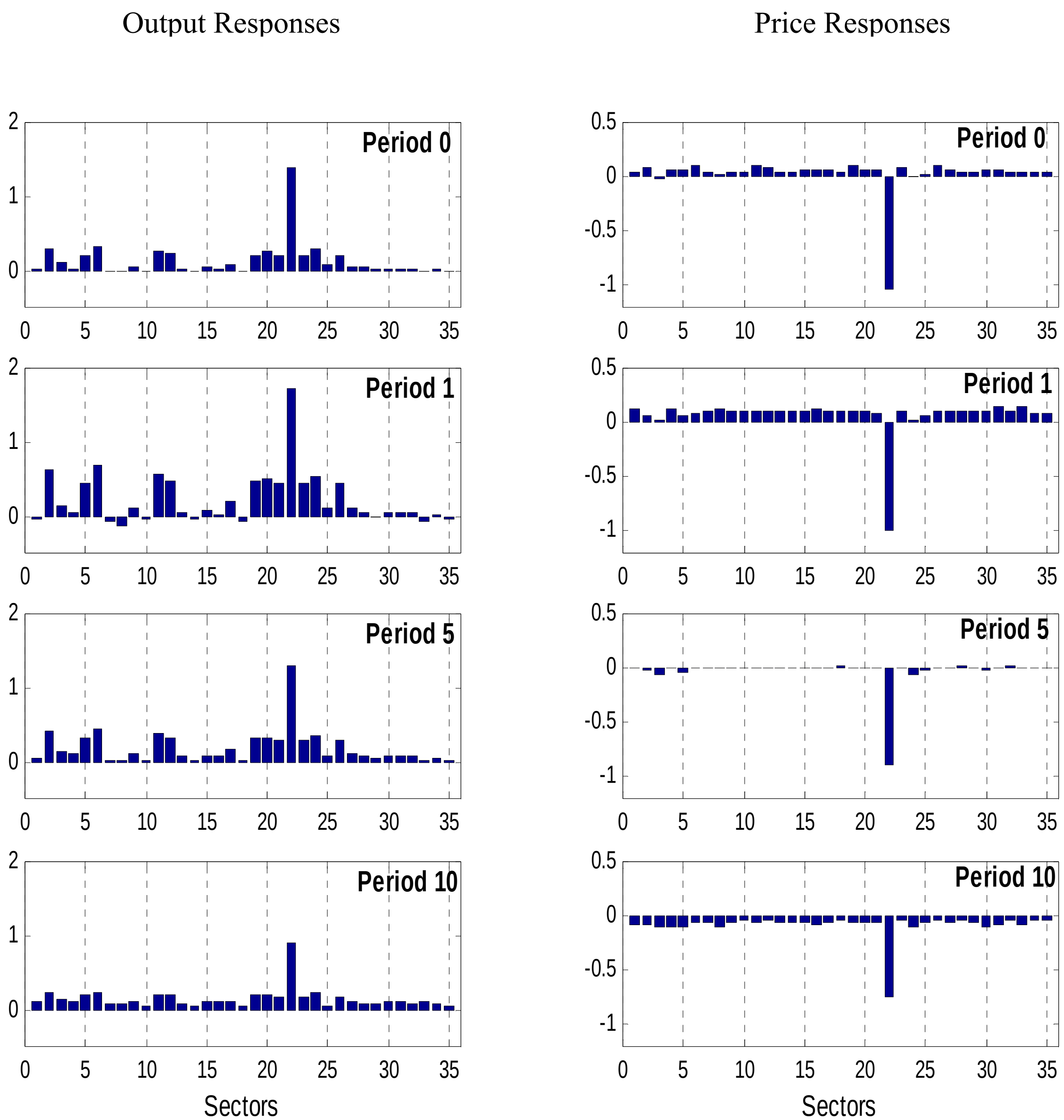
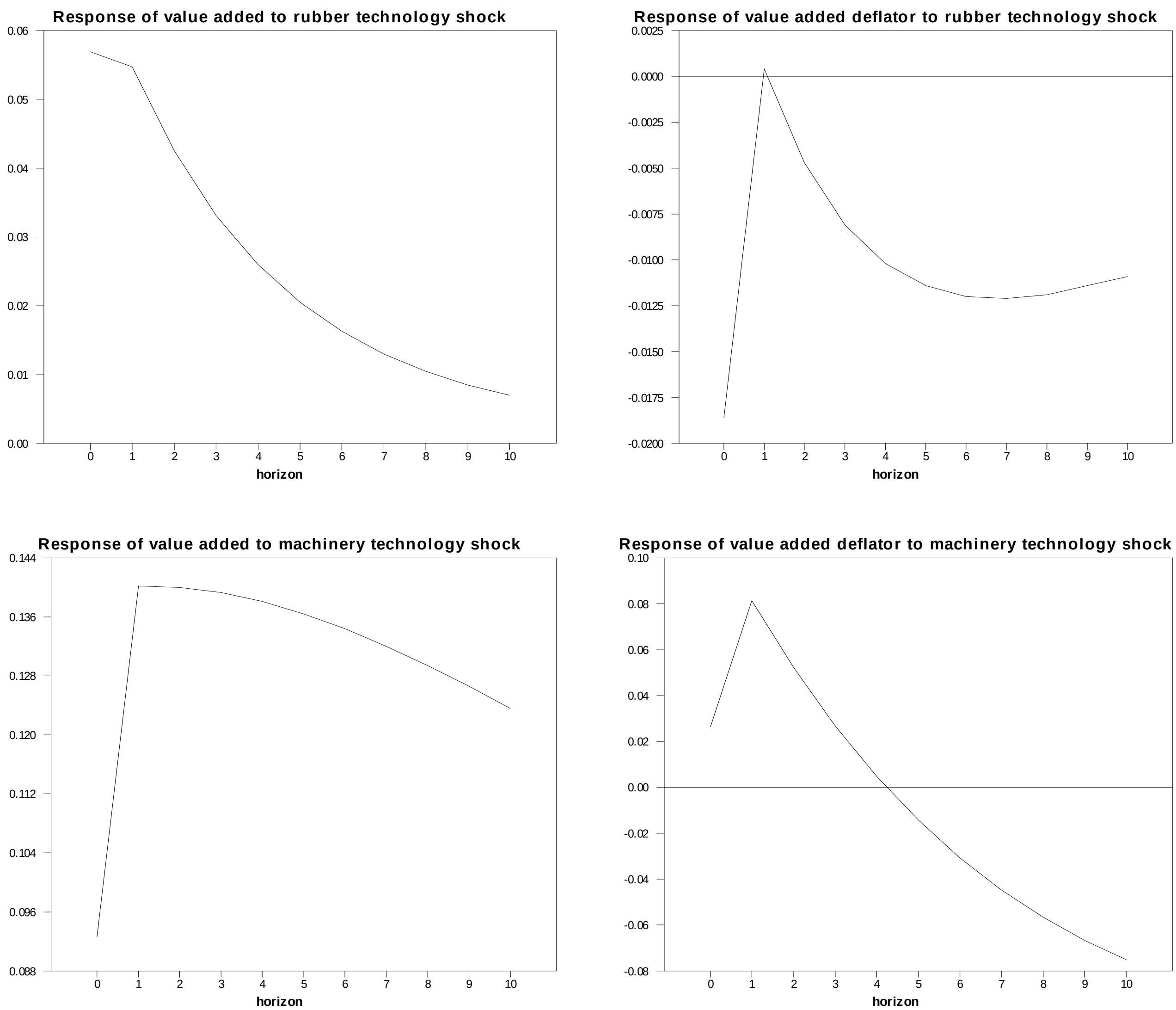


Figure 9. Response of Aggregate Value Added and Value Added Deflator to Technology Shocks in the Rubber and Machinery Sectors



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