
Diminishing marginal impatience: its promises for asset pricing

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This study argues that *diminishing marginal impatience (DMI)* as an intuitively plausible behavioural assumption of endogenous time preference has the potential for resolving important issues like the *equity premium puzzle*. It shows that, while applied to a model in the traditional overlapping generations (*OG*) framework, *DMI* is capable of generating assets prices with magnitude and volatility higher than those suggested by standard models with *constant marginal impatience (CMI)*.

I. Introduction

There has been some renewed interest in the concept of endogenous time preference as an intuitively appealing behavioural assumption in recent growth literature.¹ In this stream of literature, the rate of time preference – also known as ‘marginal impatience’ – that captures the willingness to postpone consumption at the margin, has been shown to depend on, among other things, current consumption. Most previous works relating to endogenous time preference typically assume *increasing marginal impatience (IMI)* hereafter, implying that people are less patient at higher levels of consumption than at lower levels. This, however, seems to be counter-intuitive. In contrast, the assumption of *diminishing*

marginal impatience (DMI) hereafter) appears to be intuitively more plausible.² Although recent studies shed sufficient light on the theoretical relevance and empirical validity of *DMI*, the significance of this behavioural assumption in applications – where standard models do not usually perform well – has hardly been analysed.³ This letter focuses on the application of *DMI* to asset pricing, thus exploring its potential for resolving issues related to the celebrated equity premium puzzle (*a la* Mehra and Prescott, 1985). It argues that in a consumption-based capital asset pricing model (CCAPM) preferences characterized by *DMI* (henceforth abbreviated as *DMI*-preferences) may generate higher returns to risky assets as well as higher volatility of their prices as compared with those suggested by the

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¹For a brief survey, see Das (2003).

²Koopmans (1986, pp. 94–5) questions this assumption of *IMI* on intuitive grounds and writes to the effect that the opposite assumption of decreasing marginal impatience is likely to be the ‘normal case’. According to Barro and Sala-I-Martin (1995, p. 109), the assumption of *IMI* is ‘unappealing’ because ‘it is counterintuitive that people would raise their rates of time preference as the level of consumption rises’. For a discussion on intuitive appeal of *DMI*, see Fisher (1930, p. 72).

³See Becker and Mulligan (1997) for theoretical justification. Lawrance (1991), Ogaki and Atkeson (1997) and Samwick (1998) provide rich empirical support in favour of *DMI*. Throughout, the study refers to ‘standard models’ as those with constant (‘exogenous’ to contrast ‘endogenous’) time preference.

standard models. Given the fact that these are two important aspects of the failure of standard models in resolving the puzzle indicates that *DMI* may hold promises for understanding the equity premium puzzle.⁴

DMI is incorporated in a standard overlapping generations (henceforth, *OG*) framework with a single asset. Individuals' rate of time preference is modelled as a decreasing function of their current consumption levels.⁵ This innovation in preference pattern gives rise to a time-non-separable intertemporal felicity function that has major implications for individual risk aversion and intertemporal elasticity of substitution. These, in turn, have important bearings on mean and variance of asset prices.

The rest of the paper is organized as follows. Section II outlines the model, explicitly derive, and compares the returns from a risky asset to those derived from a comparable model with *constant marginal impatience* (*CMI* hereafter). Section III includes a brief discussion of the results and concluding remarks.

II. Asset Pricing with *DMI*

Outline of the model

Consider an *OG* model in which a single, two-period-lived agent is born in each period. The agent maximizes her expected lifetime utility represented by: $u(c_t) + \phi(c_t)v(c_{t+1})$, where c_t denotes her consumption in period t ; $u(\cdot)$ and $v(\cdot)$ denote period utilities in period t and $t+1$ respectively; and $\phi(\cdot)$ is the endogenous discount factor. Note that $\phi(c_t) = 1/\rho(c_t) - 1$, where $\rho(c_t)$ is the endogenous rate of time preference. The periodic endowment of an agent born in period t , is given by the set $\{y_t, 0\}$. The realization of y_t occurs prior to the appearance of generation $(t+1)$.

Following Huffman (1986), it is assumed that y_t is i.i.d. with the distribution function $G(Y)$. In the first period of her life an agent optimally allocates her first period endowment on current consumption and savings. Savings take the form of purchasing perfectly durable and non-producible capital at price p_t per unit, which can be sold in the last period of her life for a price p_{t+1} .⁶ In addition, the agent collects an exogenous constant dividend

r yielded per unit of capital (paid in terms of the consumption good). Thus the problem facing a young agent at time t is:

$$\begin{aligned} & \text{Max}_{\{z\}} E_t \{u(c_t) + \phi(c_t)v(c_{t+1})\} \\ & \text{subject to (i) } c_t = y_t - p_t z \quad \text{and} \\ & \text{(ii) } c_{t+1} = (p_{t+1} + r)z \end{aligned} \quad (1)$$

where z denotes the number of units of capital in the economy, which is normalized to unity in equilibrium.

The period utility functions $u(\cdot)$ and $v(\cdot)$ are increasing, strictly concave, twice continuously differentiable and satisfy the *Inada* conditions. Individual preferences satisfy *DMI* in the sense that their subjective valuation of the future utility (the discount factor) is increasing in current consumption, and is duly characterized by $\phi' > 0$. Further, one assumes $\phi > 0$ and $\phi'' < 0$ for all $c_t > 0$.

Equation 1 implies that the expected utility function is not additively time-separable, and marginal utility of future consumption increases with current consumption. What it means for the *intertemporal elasticity of substitution* (*IES*) is formalized in the following Lemma.

Lemma 1: *With *DMI*, the *IES* is lower compared to when time preference is constant (*CMI*).*

Proof: Suppose $p_t = 1$ and individuals choose z . The first order condition of the above maximization problem can then be written as

$$-u(c_t) - \phi'(c_t)E_t\{v(c_{t+1})\} + \phi(c_t)E_t\{v'(c_{t+1})R_{t+1}\} = 0$$

or,

$$-u(c_t) - \phi'(c_t)E_t\{v(x_{t+1}c_t)\} + \phi(c_t)E_t\{v'(x_{t+1}c_t)R_{t+1}\} = 0$$

where, $x_{t+1} = c_{t+1}/c_t$ and $R_{t+1} = p_{t+1} + r$. Differentiating the above equation with respect to R_{t+1} and x_{t+1} and rearranging yields,

$$\begin{aligned} \frac{dx_{t+1}}{x_{t+1}} &= \left(\frac{\phi'(c_t)E_t\{v(c_{t+1})\}}{\phi(c_t)R_{t+1}} - \frac{E_t\{v''(c_{t+1})c_{t+1}\}}{E_t\{v'(c_{t+1})\}} \right)^{-1} \frac{dR_{t+1}}{R_{t+1}} \\ &= \hat{\sigma}(c_t, c_{t+1}, R_{t+1}) \frac{dR_{t+1}}{R_{t+1}}, \end{aligned}$$

⁴This article should not be interpreted as an attempt to resolve the equity premium puzzle which requires a more complete and elaborate specification of the model which may involve complexities.

⁵See Chakrabarty (2002) and Das (2003) for models that utilize this *DMI* specification.

⁶This capital is alternatively referred to as 'equity' or 'asset'.

where $\widehat{\sigma}(c_t, c_{t+1}, R_{t+1})$ is the ‘modified’ *IES*.⁷ With *CMI* (i.e., $\phi'_t = 0$), the *IES* is given by $\sigma(c_{t+1})$, satisfying

$$\frac{dx_{t+1}}{x_{t+1}} = \left(-\frac{E_t\{v''(c_{t+1})c_{t+1}\}}{E_t\{v'(c_{t+1})\}} \right)^{-1} \frac{dR_{t+1}}{R_{t+1}} = \sigma(c_{t+1}) \frac{dR_{t+1}}{R_{t+1}}$$

A comparison between $\sigma(c_{t+1})$ and $\widehat{\sigma}(c_t, c_{t+1}, R_{t+1})$ readily reveals that $\sigma(c_{t+1}) > \widehat{\sigma}(c_t, c_{t+1}, R_{t+1})$ for $c_t > 0$, $c_{t+1} > 0$ and $R_{t+1} > 0$. QED

Equilibrium asset prices

Even though the dividend r is assumed to be constant, the endowment y_t has a distribution $g(y_t)$ with $y_t \in (0, \infty)$. Aggregate consumption in period t is given by $(y_t + r)$. Under the assumptions of rational expectations and of stationary equilibria, it is easy to show that in equilibrium both consumption and price depend on current realization of Y . Note that consumption being a normal good by assumption, the consumption function is strictly increasing in y_t . Furthermore, the pricing function $p_t = p(y_t)$, is strictly increasing and differentiable in y_t .

With $z_t = 1$ the first-order condition writes

$$\begin{aligned} p(y_t)(u'(c_t) + \phi'(c_t)E_t\{v(c_{t+1})\}) \\ = \phi(c_t)E_t\{v'(c_{t+1})(p(y_{t+1}) + r)\} \end{aligned} \quad (2)$$

We simplify notation by denoting $u_t = u(c_t)$, $u'_t = u'(c_t)$, $v_t = v(c_t)$, $v'_t = v'(c_t)$, $\phi_t = \phi(c_t)$ and $\phi'_t = \phi'(c_t)$. Rewriting Equation 2, one has:

$$\begin{aligned} p(y_t)(u'_t + \phi'_t E_t\{v_{t+1}\}) &= \phi_t E_t\{v'_{t+1}(p(y_{t+1}) + r)\} \\ &= \phi_t (E_t\{v'_{t+1}\} E_t\{p(y_{t+1}) + r\} \\ &\quad + Cov_t(v'_{t+1}, p(y_{t+1}) + r)) \end{aligned}$$

Solving for expected return on equity, one obtains:

$$\begin{aligned} E_t\{p(y_{t+1}) + r\} \\ = \frac{p(y_t)(u'_t + \phi'_t E_t\{v_{t+1}\}) - \phi_t Cov[v'_{t+1}, p(y_{t+1}) + r]}{\phi_t E_t\{v_{t+1}\}} \end{aligned} \quad (3)$$

For the standard model with *CMI* $\phi_t = \bar{\phi} \forall t$, ($\phi'_t = 0$), the expected future asset prices are simply $p(y_t)u'_t - \bar{\phi} Cov[v'_{t+1}, p(y_{t+1}) + r] / \bar{\phi} E_t\{v_{t+1}\} < RHS$ of Equation 3. Note that because of the assumption of diminishing marginal periodic utility, the covariance term is always negative.

Equation 3 is the main result of this study which says: (a) returns from equities are greater if preferences satisfy *DMI* and (b) expected returns

are increasing in the level of marginal impatience (measured by $\rho = 1/(1 + \phi)$) on the part of young agents. The intuition behind this result is simple. In the present model, an increase in current consumption of an agent serves dual purposes: it raises her current utility, and more importantly, it, by making her less impatient (alternatively, more patient), increases her appreciation for future utility. The latter effect is responsible for a lower *IES* compared to that in the standard model (Lemma 1). A lower *IES*, in turn, implies that intertemporal substitution of current consumption by future consumption requires higher returns on assets.

Volatility of asset prices

It is difficult to compare volatility of asset prices generated by two distinct assumptions about time preference within the general specification of the above models. Therefore, in the following example, specific functional forms of period utilities and discount factor are utilized, and the models are simulated for specific parameter values to demonstrate that *DMI*-preference generates higher volatility of asset prices for all values of $y_t \in (0, \infty)$.⁸

Let the period utility functions be given by $u(c) = v(c) = \log(c)$ and the discount factor is given by $\phi(c) = \gamma \log(c)$. The price function is implicitly given by the first order condition (Equation 2) as:

$$p(y_t)(1 + \gamma \lambda_t) - \gamma(y_t - p(y_t)) \log(y_t - p(y_t)) = 0 \quad (4)$$

where $\lambda_t = E_t\{\log(p(y_{t+1}) + r)\}$. In the standard *OG* model with *CMI*, the equilibrium price function is given by

$$p(y_t) = \frac{\bar{\phi} y_t}{1 + \phi} \quad (5)$$

The values of $p(y)$ in both cases are found by numerical simulations using Newton’s convergence method. As shown in Table 1, mean and variance of asset prices generated under standard model specification are less than those under *DMI*-preference specification. Thus endogenous time preference (*DMI*) can account for the observed higher variability of asset prices that most standard neoclassical models fail to generate.

III. Discussion and Conclusion

Using an *OG* framework this study demonstrates how *DMI* as an intuitive and empirically relevant

⁷The qualifier ‘modified’ is used to contrast it with the *IES* derived from the standard model with *CMI*.

⁸The Matlab program for this simulation exercise is available from the author upon request.

Table 1. Mean and variances of asset returns

	Model with <i>DMI</i> (Equation 4)	Standard model with <i>CMI</i> (Equation 5)
Mean	1.23 (1.15, 1.32)	0.62 (0.59, 0.66)
Variance	0.05	0.02

Notes: The parameter values used are: $\bar{\phi}=0.22$ (with an assumption of 1 period=30 years in the OG framework, this implies an annual discount rate of 0.95), $\gamma=0.95$ and $r=0.2$. However, the numerical results hold for a wide range of parameter values. Means and variances have been calculated from 1000 numerical simulations. The figures in parentheses denote the values at the 95% confidence interval.

form of time preference, has the potential of explaining the equity premium puzzle. This possibility emerges from the results that *DMI*-preferences not only lead to higher returns for risky assets but also generate higher volatility of these returns. The crucial mechanism at work is that the time-nonseparability of preferences built into the model assigns dual roles to the period 1 consumption of an agent: it directly affects current utility and through the endogenous rate of time preference it lowers the *IES* between two periods.

Instead of imposing time-non-separability as an ad-hoc assumption, *DMI* provides an intuitive justification for allowing non-separable preferences. Mehra and Prescott (1985) strongly advocate the use of time-non-separability as an alternative, as they put the blame on the rigid additively time-separable specification of preferences for the dramatic failure of standard models to generate the observed equity premium. In the present model, because of this time-non-separability of preference the volatility of asset prices is affected by the volatility of endowment (income) not only through current utility but also through the endogenous discount factor. In contrast, standard models with time-separable preference provide only one channel through current utility. Consequently, one observes higher volatility of asset prices generated by *DMI*-preferences.

DMI may be useful in resolving the equity premium puzzle through yet another channel. Preferences involving high risk aversion would require high risk premium that could potentially generate high returns to risky assets. With *DMI*-preferences under the current model set-up, this channel, however, requires additional restrictions on the specification of time

preference and it is outside the scope of this short letter.

The entire analysis is based on the assumption of a single asset with uncertain return. Does the model fit into a more realistic multi-asset framework? Can this model explain other aspects of the equity premium puzzle, viz. the risk-free rate puzzle?⁹ Admittedly, the model, in its present form, is not capable of delivering definitive answers to these questions. Bringing in other features such as heterogeneity of preferences across individuals and market imperfection lays out the agenda for future research.

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References

- Barro, R. and Sala-I-Martin, X. (1995) *Economic Growth*, McGraw-Hill, New York.
- Becker, G. S. and Mulligan, C. B. (1997) The endogenous determination of time preference, *Quarterly Journal of Economics*, **112**, 729–58.
- Chakrabarty, D. (2002) Poverty traps and growth in a model of endogenous time preference, Center for European Integration Studies Working Paper.
- Das, M. (2003) Optimal growth with decreasing marginal impatience, *Journal of Economic Dynamics and Control*, **27**, 1881–98.
- Fisher, I. (1930) *The Theory of Interest*, Macmillan, New York.
- Huffman, G. W. (1986) The representative agent, overlapping generations and asset pricing, *Canadian Journal of Economics*, **19**, 511–21.
- Koopmans, T. C. (1986) Representation of preference ordering over time, in *Decision and Organization*, 2nd edn (Eds) C. B. McGuire and R. Radner, University of Minnesota Press, Minneapolis, MN.
- Lawrance, E. C. (1991) Poverty and the rate of time preference: evidence from panel data, *Journal of Political Economy*, **99**, 54–75.
- Mehra, R. and Prescott, E. C. (1985) The equity premium: a puzzle, *Journal of Monetary Economics*, **15**, 145–61.
- Ogaki, M. and Atkeson, A. (1997) Rate of time preference, *IES*, and level of wealth, *Review of Economics and Statistics*, **79**, 564–72.
- Samwick, A. A. (1998) Discount rate homogeneity and social security reform, *Journal of Development Economics*, **57**, 117–46.
- Weil, P. (1989) The equity premium puzzle and the risk-free rate puzzle, *Journal of Monetary Economics*, **24**, 401–21.

⁹ High risk aversion required to ‘solve’ the equity premium puzzle implies that the real interest rate should be high for a rich economy. However, as pointed out by Weil (1989) the real interest rate has been scarcely positive over long periods. This is called the ‘risk-free rate puzzle’.