

LOADED DICE AND CENTRAL TENDENCY: ANALYZING GAMES OF CHANCE

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OVERVIEW

- ③ I describe an activity I've used with middle school students and prospective elementary teachers
- ③ Play a game
- ③ Analyze the game
- ③ Links to probability & data analysis



RATIONALE & CONTENT

- ◎ It has become increasingly important that students understand probability and are able to make sense of data. (NCTM, American Statistical Association, Common Core State Standards)
- ◎ Anticipating variability, accounting for variability, describing distributions

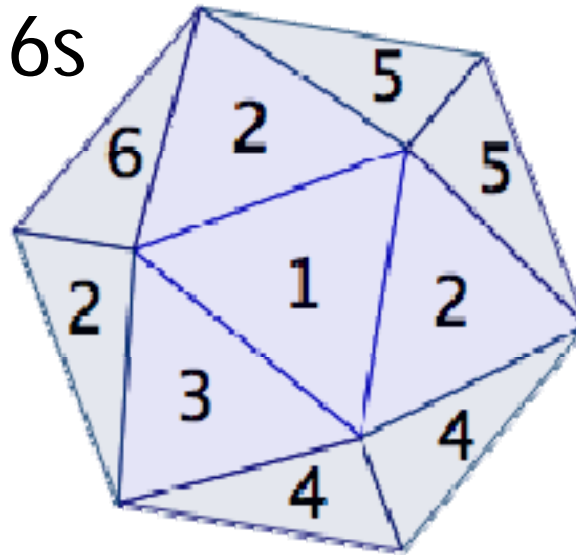


THE GAME

- ③ Students are given a special die.
- ③ Predict the outcome of a single roll
- ③ Roll the die and record the outcome
- ③ Determine the score based on certain rules

SPECIALLY-MARKED ICOSAHEDRAL DIE

- © 20 faces marked with seven 1s, four 2s, one 3, two 4s, two 5s, and four 6s



1 1 1 1 1 1 1 2 2 2 2 3 4 4 5 5 6 6 6 6

SCORING METHOD 1

Comparison of Outcome & Prediction	Points
Match	100
Don't match	10

SCORING METHOD 2

Outcome v. Prediction	Points
Match	55
Difference is 1	45
Difference is 2	35
Difference is 3	25
Difference is 4	15
Difference is 5	5

$$\text{Points} = 55 - 10|\text{Outcome} - \text{Prediction}|$$

PLAYING THE GAME

- ◎ Play with 10 rolls using Scoring Method 1.
- ◎ Calculate score and compare with others.
- ◎ Share thoughts on strategies for winning.



PLAYING THE GAME

- ③ Play with 10 rolls using scoring method 2
 - ③ Calculate score and compare with others.
 - ③ Share thoughts on strategies for winning.
 - ③ Compare to Scoring Method 1.

ANALYZING THE GAME

◎ Roll the die 30 times and record outcomes.

◎ Example:

outcome	1	2	3	4	5	6
frequency	9	5	3	3	5	5

UNDER SCORING METHOD 1

- ⊙ What are maximum and minimum possible scores?
- ⊙ Pretend we predicted 1 each time. What would the score be?
- ⊙ Repeat for each possible prediction.

UNDER SCORING METHOD 1

◎ Example

outcome	1	2	3	4	5	6
frequency	9	5	3	3	5	5

◎ If I had predicted 1 each time, I would have scored 1110 points.

◎ $9(100)+5(10)+3(10)+3(10)+5(10)+5(10)$

◎ $9(100)+21(10)$

UNDER SCORING METHOD 1

◎ Example

outcome	1	2	3	4	5	6
frequency	9	5	3	3	5	5

Prediction each time	1	2	3	4	5	6
Potential Score	1100	750	570	570	750	750



"BEST PREDICTIONS"

- ◎ Students use numerical reasoning to help calculate the scores.
- ◎ Students identify the "best prediction" for their set of outcomes.
- ◎ The class pools data to see the most common (modal) "best prediction."



UNDER SCORING METHOD 2

- ⊙ What are maximum and minimum possible scores?
- ⊙ Pretend we predicted 1 each time. What would the score be?
- ⊙ Repeat for each possible prediction.

UNDER SCORING METHOD 2

◎ Example

outcome	1	2	3	4	5	6
frequency	9	5	3	3	5	5

◎ If I had predicted 1 each time, I would have scored 1000 points.

◎ $55(9)+45(5)+35(3)+25(3)+15(5)+5(5)$

UNDER SCORING METHOD 2

If I always predicted	then Scoring Method 2 would give this many points					
	# of 1s	# of 2s	# of 3s	# of 4s	# of 5s	# of 6s
1	55()	+45()	+35()	+25()	+15()	+ 5() =
2	45()	+55()	+45()	+35()	+25()	+15() =
3	35()	+45()	+55()	+45()	+35()	+25() =
4	25()	+35()	+45()	+55()	+45()	+35() =
5	15()	+25()	+35()	+45()	+55()	+45() =
6	5()	+15()	+25()	+35()	+45()	+55() =

UNDER SCORING METHOD 2

$$\begin{bmatrix} 55 & 45 & 35 & 25 & 15 & 5 \\ 45 & 55 & 45 & 35 & 25 & 15 \\ 35 & 45 & 55 & 45 & 35 & 25 \\ 25 & 35 & 45 & 55 & 45 & 35 \\ 15 & 25 & 35 & 45 & 55 & 45 \\ 5 & 15 & 25 & 35 & 45 & 55 \end{bmatrix} \times \begin{bmatrix} \# \text{ of } 1\text{s} \\ \# \text{ of } 2\text{s} \\ \# \text{ of } 3\text{s} \\ \# \text{ of } 4\text{s} \\ \# \text{ of } 5\text{s} \\ \# \text{ of } 6\text{s} \end{bmatrix}$$



"BEST PREDICTIONS"

- ◎ Students identify the "best prediction" for their set of outcomes.
- ◎ Pool data to see the most common (modal) "best prediction."



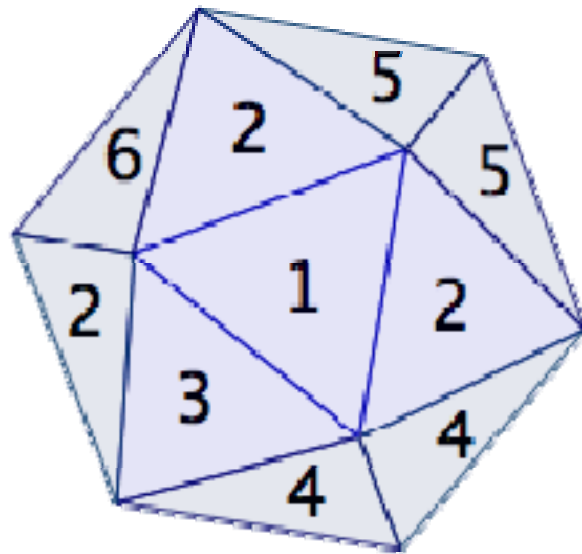
STUDENTS' REASONING

- ◎ Students discuss why these numbers are the "best" for the two scoring methods.
- ◎ Link to measures of center of set of possible outcomes.
- ◎ Link to properties of mode and median.

SPECIALLY-MARKED ICOSAHEDRAL DIE

◎ The sample space is

1 1 1 1 1 1 1 2 2 2 2 3 4 4 5 5 6 6 6 6



Mode = 1
Median = 2
Mean = 3

EXTENSION

- ◎ A scoring method highlighting the mean is based on squared differences

Absolute difference of outcome & prediction	0	1	2	3	4	5
Points	50	48	42	32	18	0

$$\text{Points} = 50 - 2(\text{Outcome} - \text{Prediction})^2$$



OTHER WAYS OF PLAYING

- ◎ Strips of paper
- ◎ TI-83/84 Prob Sim App
- ◎ Write your own simulation with TI-Nspire, Fathom, Excel, or Geogebra

TI PROB SIM APP

- ◎ Roll Dice – Set Sides = 20
Under “ADV,” change weights for 1 to 6 to match the frequencies, and change weights for 7 to 20 to be 0.
- ◎ Roll Dice – Set Sides = 6
Under “ADV,” change weights for 1 to 6 to match the frequencies



GEOGEBRA SIMULATION

- © Available online
- © www.shsu.edu/~dlj006/mygggb/icosahedron.html

CLOSING

This activity encourages students to

- ⊙ anticipate and account for variability
- ⊙ use numerical reasoning
- ⊙ apply knowledge of probability
- ⊙ reason about measures of center



QUESTIONS? COMMENTS?

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- © www.shsu.edu/~dlj006/pres.html