1. Graph the functions below and decide if they are even, odd, or neither even nor odd.

(a) \( f(x) = 3x^4 + 3 \)

(b) \( f(x) = 2x^3 - x \)

(c) \( f(x) = 2x^3 - x + 2 \)

(d) \( f(x) = \frac{1}{x^2 + 1} \)

(e) \( f(x) = \frac{x}{x^2 + 1} \)

**Solutions.**

(a) \( f(x) = 3x^4 + 3 \) is even.

(b) \( f(x) = 2x^3 - x \) is odd.

(c) \( f(x) = 2x^3 - x + 2 \) is neither even nor odd.

(d) \( f(x) = \frac{1}{x^2 + 1} \) is even. Here is the graph:

![Graph of f(x) = 1/(x^2 + 1)](image)

(e) \( f(x) = \frac{x}{x^2 + 1} \) is odd. Here is the graph:

![Graph of f(x) = x/(x^2 + 1)](image)
2. You are given the graphs of certain functions. Determine if the function is even, odd, or neither.

Solutions.
(a) Even
(b) Odd
(c) Odd
(d) Even
3. Decide algebraically if the function is even, odd, or neither.

(a) \( f(x) = x^3 - 4x \)
(b) \( f(x) = \frac{x}{1 + x^2} \)
(c) \( f(x) = x^5 + 7x^2 - 3x + 5 \)

**Solution.**

(a) The function \( f(x) = x^3 - 4x \) is odd since it is symmetric about the origin. We can check this algebraically:

\[
 f(-x) = (-x)^3 - 4(-x) = -x^3 + 4 = -(x^3 - 4) = -f(x). 
\]

(This is done in the lecture notes.)

(b) If \( f(x) = \frac{x}{1 + x^2} \) then \( f(-x) = \frac{-x}{1 + (-x)^2} \). Since \( (-x)^2 = x^2 \) we can simplify this to

\[
 f(-x) = \frac{-x}{1 + (-x)^2} = -\frac{x}{1 + x^2} = -f(x). 
\]

So \( f(x) \) is odd. (This is done in the lecture notes.)

(c) If \( f(x) = x^5 + 7x^2 - 3x + 5 \) then

\[
 f(-x) = (-x)^5 + 7(-x)^2 - 3(-x) + 5 = -x^5 + 7x^2 + 3x + 5. 
\]

Since \( f(-x) = -x^5 + 7x^2 + 3x + 5 \) is neither equal to \( f(x) \) nor equal to \(-f(x)\) then \( f(x) \) is neither even nor odd. (This is done in the lecture notes.)

4. Look carefully at the graphs of the following periodic functions and estimate their period.

![Graphs of periodic functions](a.png)  ![Graphs of periodic functions](b.png)  ![Graphs of periodic functions](c.png)  ![Graphs of periodic functions](d.png)
Solutions.

(a) The period is $2\pi$, slightly more than 6.
(b) The period is 2.
(c) The period is $2\pi$, slightly more than 6.
(d) The period is $\pi$, slightly more than 3.
(e) The period is 2.

5. There is a function which is both even and odd! What is it?

Solution. If $f(x) = 0$ then the graph of $y = 0$ is just the $x$-axis. This has both odd and even symmetry!