In this lesson we discuss periodic functions and also introduce the greatest integer function.

Some graphs have translation symmetry, that is, we may shift the graph along the \( x \)-axis a certain amount and leave the graph unchanged. In this case the function is periodic; there is a real number \( c \) so that if we shift the graph to the left by \( c \) units, then the graph is unchanged.

Algebraically, we write \( f(x + c) = f(x) \).

The smallest positive real number \( c \) such that \( f(x + c) = f(x) \) is called the period of the function \( f \).

We will see this phenomenon (periodic functions and translation symmetry) throughout our study of trigonometry.
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The smallest positive real number $c$ such that $f(x + c) = f(x)$ is called the **period** of the function $f$.

We will see this phenomenon (periodic functions and translation symmetry) throughout our study of trigonometry.
Visualizing functions

For example, if we look at the graphs below, we see graphs that appear to represent periodic functions.

The graph on the left has period $2\pi$, slightly more than 6. The graph on the right has period 2.
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We digress from our discussion of periodic functions to introduce a function common in mathematics and computer science.

The greatest integer function

\[ f(x) = \lfloor x \rfloor, \]

takes as input a real number and rounds the number down to the greatest integer less than or equal to it.

For example, it rounds 3.1 to 3, so \( \lfloor 3.1 \rfloor = 3 \).

If the input is already an integer, the output is unchanged. For example, \( \lfloor 5 \rfloor = 5 \).

If the number \( x \) is positive, \( \lfloor x \rfloor \) is essentially the value of \( x \) with everything to the right of the decimal place stripped away.

So it is easy to compute \( \lfloor x \rfloor \) when \( x \geq 0 \).

One has to be careful if \( x \) is negative – we always round down here, so \( \lfloor -1.1 \rfloor = -2 \).
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Visualizing functions

Here is a graph of the greatest-integer function.
The greatest-integer function is also called the floor function since it rounds down to the integer “on the floor”, below $x$.

Notice a certain symmetry of this function: if we translate the graph up and to the right (at an angle of 45°) then we get the same graph back.

In other words, if $f(x) = \lfloor x \rfloor$ then $f(x) = f(x - 1) + 1$. 

![Graph of the greatest-integer function with points at (-2, 0), (-1, 0), (0, 0), (1, 1), (2, 2), and (3, 2).]
Visualizing functions

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Visualizing functions

A function related to the greatest-integer function is the **fractional-part function**.

The floor function throws away the decimal part of a positive real number.

What if, instead, we keep only the decimal part?

The fractional-part function $g(x) = x - \lfloor x \rfloor$ keeps just the remainder, after we remove the integer part.
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The fractional-part function is an example of a *sawtooth function* – it is periodic with very sharp edges!
Visualizing functions

(END)