On metric dimension, strong metric dimension, and zero forcing number of graphs

Eunjeong Yi
Texas A&M University at Galveston

Abstract: Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The *metric dimension* $dim(G)$ of a graph G is the minimum number of vertices such that every vertex of G is uniquely determined by its vector of distances to the chosen vertices. The *strong metric dimension* $sdim(G)$ of a graph G is the minimum among cardinalities of all strong resolving sets: $W \subseteq V(G)$ is a strong resolving set of G if for any $u, v \in V(G)$, there exists an $x \in W$ such that either u lies on an $x - v$ geodesic or v lies on an $x - u$ geodesic. The *zero forcing number* $Z(G)$ of a graph G is the minimum cardinality of a set S of black vertices (whereas vertices in $V(G) - S$ are colored white) such that $V(G)$ is turned black after finitely many applications of "the color-change rule": a white vertex is converted black if it is the only white neighbor of a black vertex.

In this talk, we show that $dim(T) \leq Z(T) \leq sdim(T)$ for a tree T and that $dim(G) - 1 \leq Z(G) \leq sdim(G)$ for a unicyclic graph G. Further, we show that $Z(G) \leq sdim(G) + 3r(G)$ for a connected graph G, where $r(G)$ is the cycle rank of G. We also discuss the effect of vertex or edge deletion on the metric dimension of graphs. We end with a proof of $dim(T) - 2 \leq dim(T + e) \leq dim(T) + 1$ for any tree T and an edge $e \in E(\overline{T})$.

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