On Writing a M.S. in Mathematics Thesis at Sam Houston State University

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DEDICATION

A dedication statement goes here.

ABSTRACT

García-Puente, Luis D., *On writing a M.S. in mathematics thesis at Sam Houston State University*. Master of Arts Department of Mathematics and Statistics, April, 2012, Sam Houston State University, Huntsville, Texas.

This is the format for the bibliographic information required for the abstract. The content of the abstract that follows this citation will vary according to the subject area. The abstract should be concise and informative; however, the abstract must be less than 350 words in length. In general, it should state the purpose and describe the subjects and the methodology used in the study. The abstract should also describe the findings, conclusions, and implications of the study. Students should consult their thesis director and style manual to determine the content of the abstract. The abstract must be signed by the thesis director below.

A list of key words must be included at the bottom of the abstract; however, key words and the title information do not count toward the 350 word total. Key words should be specific terms or phrases used in the thesis that would enable a person to successfully search out the content of the document if it were in a library database. The first word of each key term should be capitalized.

KEY WORDS: Thesis guidelines, Index word, Sam Houston State University, Graduate School, Texas.

ACKNOWLEDGEMENTS

Here you can write some nice words.

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CHAPTER 1

AMS MONOGRAPH SERIES SAMPLE

1.1 This Is A Numbered First-Level Section Head

This is an example of a numbered first-level heading.

1.1.1 This Is A Numbered Second-Level Section Head

This is an example of a numbered second-level heading.

Lemma 1. *Let $f, g \in A(X)$ and let E, F be cozero sets in X.*

1. *If f is E-regular and $F \subseteq E$, then f is F-regular.*

2. *If f is E-regular and F-regular, then f is $E \cup F$-regular.*

3. *If $f(x) \geq c > 0$ for all $x \in E$, then f is E-regular.*

The following is an example of a proof, but before a small footnote. [1]

Proof. Set $j(\nu) = \max(I \backslash a(\nu)) - 1$. Then we have

$$\sum_{i \notin a(\nu)} t_i \sim t_{j(\nu)+1} = \prod_{j=0}^{j(\nu)} (t_{j+1}/t_j).$$

Hence we have [2]

$$\begin{aligned}\prod_{\nu} \Bigg(\sum_{i \notin a(\nu)} t_i \Bigg)^{|a(\nu-1)|-|a(\nu)|} &\sim \prod_{\nu} \prod_{j=0}^{j(\nu)} (t_{j+1}/t_j)^{|a(\nu-1)|-|a(\nu)|} \\ &= \prod_{j \geq 0} (t_{j+1}/t_j)^{\sum_{j(\nu) \geq j} (|a(\nu-1)|-|a(\nu)|)}.\end{aligned} \tag{1}$$

[1] Here is an example of a footnote. Notice that this footnote text is running on so that it can stand as an example of how a footnote with separate paragraphs should be written.

[2] And here is the beginning of the second footnote.

By definition, we have $a(\nu(j)) \supset c(j)$. Hence, $|c(j)| = n - j$ implies (5.4). If $c(j) \notin a$, $a(\nu(j))c(j)$ and hence we have (5.5). $\square$

> This is an example of an 'extract'. The magnetization M_0 of the Ising model is related to the local state probability $P(a) : M_0 = P(1) - P(-1)$. The equivalences are shown in Table 1.

Table 1: Every table must have a caption.

	$-\infty$	$+\infty$
$f_+(x,k)$	$e^{\sqrt{-1}kx} + s_{12}(k)e^{-\sqrt{-1}kx}$	$s_{11}(k)e^{\sqrt{-1}kx}$
$f_-(x,k)$	$s_{22}(k)e^{-\sqrt{-1}kx}$	$e^{-\sqrt{-1}kx} + s_{21}(k)e^{\sqrt{-1}kx}$

Definition 2. This is an example of a 'definition' element. For $f \in A(X)$, we define

$$\mathscr{Z}(f) = \{E \in Z[X] : f \text{ is } E^c\text{-regular}\}. \tag{2}$$

Remark 3. This is an example of a 'remark' element. For $f \in A(X)$, we define

$$\mathscr{Z}(f) = \{E \in Z[X] : f \text{ is } E^c\text{-regular}\}. \tag{3}$$

Example 4. This is an example of an 'example' element. For $f \in A(X)$, we define

$$\mathscr{Z}(f) = \{E \in Z[X] : f \text{ is } E^c\text{-regular}\}. \tag{4}$$

Exercise 5. This is an example of the `xca` environment. This environment is used for exercises which occur within a section.

Figure 1: This is an example of a figure caption with text.

Figure 2: Every figure must have a caption.

Some extra text before the `xcb` head. The `xcb` environment is used for exercises that occur at the end of a chapter. Here it contains an example of a numbered list.

Here is an example of a cite. See [9].

Theorem 6. *This is an example of a theorem.*

Theorem 7 (Marcus Theorem)**.** *This is an example of a theorem with a parenthetical note in the heading.*

1.2 Some More List Types

This is an example of a bulleted list.

- $\mathscr{J}_g$ of dimension $3g-3$;
- $\mathscr{E}_g^2 = \{$Pryms of double covers of $C = \square$ with normalization of C hyperelliptic of genus $g-1\}$ of dimension $2g$;
- $\mathscr{E}_{1,g-1}^2 = \{$Pryms of double covers of $C = \square_{P1}^{H}$ with H hyperelliptic of genus $g-2\}$ of dimension $2g-1$;
- $\mathscr{P}_{t,g-t}^2$ for $2 \le t \le g/2 = \{$Pryms of double covers of $C = \square_{C''}^{C'}$ with $g(C') = t-1$ and $g(C'') = g-t-1\}$ of dimension $3g-4$.

This is an example of a ‘description’ list.

Zero case $\rho(\Phi) = \{0\}$.

Rational case $\rho(\Phi) \neq \{0\}$ and $\rho(\Phi)$ is contained in a line through 0 with rational slope.

Irrational case $\rho(\Phi) \neq \{0\}$ and $\rho(\Phi)$ is contained in a line through 0 with irrational slope.

CHAPTER 2

EMMY NOETHER

In recent years, there has been a development in mathematical pedagogy to integrate the original ideas of the mathematicians who actually developed the theory, rather than using textbooks that refer to these works. These original sources oftentimes show why and how problems were solved, and provide insight into the way of thinking of those genius minds. In the preface his book "Mathematical Expeditions," Reinhard Laubenbacher talks about using historical sources in university math classes he taught at New Mexico State University. The success of those classes was reflected in greater appreciation for the impetus in the development of the topics discussed and in providing the students with a simpler, or less abstract, approach. This concept of "learning from the masters" has led to the growth in interest to translate groundbreaking articles and books written by the most important mathematicians in history. Laubenbacher suggested that we translate Noether's seminal paper on describing what we now call Noetherian rings and modules. In this work, Emmy Noether describes the precise condition that generalizes principal ideal domains, and thereby forever changing the way mathematicians view rings and modules.

After an exhaustive search, we discovered that this foundational paper had already been translated by Colin McLarty, professor of mathematics and philosophy at Case Western Reserve University. McLarty is one of the leading experts in the works of Emmy Noether. He translated some of her more important contributions to mathematics and physics and published various articles that highlight her mathematical work. Over correspondence with McLarty, he suggested that there is a rather large number of very influential mathematicians and mathematical historians that would appreciate knowing more about Emmy Noether's contribution to classical invariant theory. This thesis is part of a larger project: translate

Noether's dissertation, verify the computations of the algebraic invariants she computed via an archaic symbolic approach, complete the computation of algebraic invariants of fourth degree forms in three variables, and connect the symbolic notation to modern techniques in algebraic invariant theory. Kung and Rota [9] said about this that "…the covariants of no nontrivial form (except for conics) in three or more variables have been fully classified, not even those of ternary quartic which persuaded Emmy Noether to quit invariant theory." This is the project I am presenting in this thesis.

Figure 3: Arabian Phoenix

In Chapter II of this paper, I will give a quick overview over Emmy Noether's life. In Chapter III, I will discuss the major differences between classical and modern invariant theory. In Chapter IV, I will take an in depth look at classical invariant theory via the well-known binary forms. In Chapter V, I look at modern tools via Sturmfels' book, and an algorithm to compute an invariant ring. Finally, Chapter VI contains the translation of Emmy Noether's doctoral dissertation.

2.1 About Emmy Noether

Amalie "Emmy" Noether was born on March 23, 1882 in Erlangen, Germany into a Jewish family. Emmy's father was the mathematician Max Noether, who lectured at the University of Erlangen. Emmy had plans to become a teacher, but instead ended up studying math at the University of Erlangen. She completed her dissertation (which has been translated here) in 1907 under Paul Gordan, and then worked for the university for seven years without getting paid. In 1915, David Hilbert and Felix Klein wanted her to join the mathematics department of the University of Göttingen, but again, the university objected to hiring a female. She taught under Hilbert's name for four years and became very well known in the mathematical community. She proved a theorem, now called "Noether's Theorem," which is one of the most important theorems in mathematics to guide and develop physics. Her theorem is the proof that "Energy may neither be created nor destroyed" and other laws of conservation. Her habilitation was finally approved in 1919, and she was allowed to teach under her own name. In 1920, Noether started working in the area of abstract algebra with Werner Schmeidler. She published papers on theory of ideals and as a result of her work on rings, the term "Noetherian Rings" was being used. She worked with B. L. Van der Waerden and was a great influence on the work he published in his book Moderne Algebra, which was one of the first central textbook in the algebra. Noether supervised multiple doctoral students, having great influence on their work. Examples of those are Max Deuring, who worked in the area of Arithmetic Geometry, and Wolfgang Krull who advanced the area of Commutative Algebra. Emmy Noether taught at the University of Göttingen until 1933, when the Prussian Ministry of Sciences, Art, and Public Education withdrew the right to teach from her. She, and a lot of other Jewish colleagues were forced to find jobs outside of Germany. Through the support of highly influential mathematicians, Emmy Noether was invited to teach at Bryn Mawr University, and also started lecturing at Princeton University in 1934, where Albert Einstein and Hermann Weyl worked. Noether died in 1935 after surgery to remove an ovarian cyst.

2.2 Things Other Great Mathematicians Said About Emmy Noether

Solomon Lefschetz from Princton University wrote to Jacob Billikopf at Bryn Mawr University about Emmy Noether:

"... she is the holder of a front rank seat in every sense of the word. As the leader of the modern algebra school, she developed in recent Germany the only school worthy of note in the sense, not only of isolated work, but of very distinguished group scientific work. In fact, it is no exaggeration to say that without exception all the better young German mathematicians are her pupils. ..."

Norbert Wiener from the Massachusetts Institute of Technology also wrote to Mr Billikopf:

" Leaving all questions of sex aside, she is one of the ten or twelve leading mathematicians of the present generation in the entire world and has founded what is certain to be the most important close-knit group of mathematicians in Germany–the Modern School of Algebraists."

Albert Einstein writes in his obituary about Emmy Noether: [15]

"In the judgment of the most competent living mathematicians, Fräulein Noether was the most significant creative mathematical genius thus far produced since the higher education of women began."

Hermann Weyl said the following at Emmy Noether's funeral:

"Justifiably proud, for you were a great woman mathematician - I have no reservations in calling you the greatest that history has known. Your work has changed the way we look at algebra, and with your many gothic letters you have left your name written indelibly across its pages. No-one, perhaps, contributed as much as you towards remoulding the axiomatic approach into a powerful research instrument, instead of a mere aid in the logical elucidation of the foundations of mathematics, as it had previously been. Amongst your predecessors in algebra and number theory it was probably Dedekind who came closest." [15]

CHAPTER 3

A BRIEF ACCOUNT OF MODERN INVARIANT THEORY VS CLASSICAL INVARIANT THEORY

3.1 Modern Invariant Theory

In modern invariant theory, we have Γ, a subgroup of $GL(\mathbb{C}^n)$. Given a polynomial $f \in \mathbb{C}[x_1,\ldots,x_n]$ then every linear transformation $\pi \in \Gamma$ transforms f into a new polynomial function $f \circ \pi$. The goal is to compute the ring of invariants for Γ:

$$\mathbb{C}[x_1,\ldots,x_n]^\Gamma := \{f \in \mathbb{C}[x_1,\ldots,x_n] : \forall \pi \in \Gamma (f = f \circ \pi)\}.$$

Example 8 (Symmetric polynomials)**.** We let S_n be the group of permutation matrices in $GL(\mathbb{C}^n)$. Its invariant ring $\mathbb{C}[x_1,\ldots,x_n]^{S_n}$ is the subring of symmetric polynomials $\mathbb{C}[\sigma_1,\ldots,\sigma_n]$, by definition of symmetric polynomials.

In classical invariant theory we consider the homogeneous polynomial (also known as a form) of degree n,

$$\mathscr{F}(x,y) = \sum_{i+k=n} c_{ik} \binom{n}{i} x^i y^k.$$

For $A \in GL_2(\mathbb{C})$, we map the variables

$$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}.$$

Then,

$$\mathscr{F}(x,y) \mapsto \mathscr{F}(x',y') = \sum_{i+k=n} c'_{ik} \binom{n}{i} x^i y^k.$$

Definition 9. An invariant of the form $\mathscr{F}$ is a polynomial $\mathscr{I}(c_{ik})$ such that

$$\mathscr{I}(c'_{ik}) = \det(A)^p \cdot \mathscr{I}(c_{ik}),$$

for some $p \in \mathbb{N}$

Example 10 (The Quadratic Binomial Case)**.** When $n = 2$, let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \in GL_2(\mathbb{C})$.

Then,

$$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}.$$

$\mathscr{F}(x,y) = c_0x^2 + 2c_1xy + c_2y^2$ becomes $\mathscr{F}(x',y') = c'_0x^2 + 2c'_1xy + c'_2y^2$.

Then one invariant is $c_0c_2 - c_1^2$, because

$$\begin{aligned} c'_0c'_2 - c'^2_1 &= c_0^2(a_{11}^2a_{12}^2 - a_{11}^2a_{12}^2) \\ &\quad + 2c_0c_1(a_{11}^2a_{12}a_{22} + a_{12}^2a_{11}a_{21} - a_{11}a_{12}(a_{11}a_{22} + a_{12}a_{21})) \\ &\quad + c_0c_2(a_{11}^2a_{22}^2 + a_{12}^2a_{21}^2 - 2a_{11}a_{12}a_{21}a_{22}) \\ &\quad + c_1^2(4a_{11}a_{21}a_{12}a_{22} - (a_{11}a_{22} + a_{12}a_{21})^2) \\ &\quad + c_2^2(a_{21}^2a_{22}^2 - a_{21}^2a_{22}^2) \\ &\quad + 2c_1c_2(a_{11}a_{21}a_{22}^2 + a_{12}a_{22}a_{21}^2 - a_{21}a_{22}(a_{11}a_{22} + a_{12}a_{21})) \\ &= c_0c_2(a_{11}a_{22} - a_{12}a_{21})^2 - c_1^2(a_{11}a_{22} - a_{12}a_{21})^2 \\ &= \delta^2(c_0c_2 - c_1^2). \end{aligned}$$

Definition 11. A covariant of the form $\mathscr{F}$ is a polynomial in the $\mathscr{C}(c_{ik},x,y)$ such that

$$\mathscr{C}(c'_{ik},x,y) = \det(A)^p \cdot \mathscr{C}(c_{ik},x,y),$$

for some $p \in \mathbb{N}$.

The power of the coefficients c_{ik} is called the degree of $\mathscr{C}$ the power of the variables x,y is called the order of $\mathscr{C}$. Note that an invariant is a covariant of order zero.

Invariant Theory was studied by a lot of great mathematicians in the late 1800s and early 1900s like Paul Gordan, Felix Klein, Hermann Weyl, Bartel L.van der Waerden, George Boole, Arthur Cayley, David Hilbert, Emmy Noether to name just a few. There had been a "British School," which Boole was part of, but later Göttingen became the "world capital of invariant theory." Paul Gordan was one of the main contributors to invariant theory. He proved the Finiteness Theorem for binary forms in 1868. In this proof Gordan constructed the invariant ring for binary forms and showed that it is finitely generated. People had tried to generalize this result, but nobody could successfully prove it until 1890 when Hilbert published his first proof of the Finiteness Theorem for general forms. It proved that there exists a finite basis for any system of invariants of arbitrary degree and with an arbitrary number of indeterminantes. Gordan did not like Hilbert's proof since it was an existence proof rather than a constructive proof. This is when Gordan said his famous, "Das ist Theologie, nicht Mathematik," which means"This is Theology, not Mathematics." Later, Hilbert published a constructive proof of the Finiteness Theorem where he gave an algorithm on how to construct the invariant ring. Gordan was much happier with this proof, and began to see the advantages of the more theoretic approach. Hilbert's constructive proof essentially "solved" the problems in classical invariant theory by producsing an algorithm to find the invariants.

CHAPTER 4

INTRODUCTION

4.1 First Section

The significance of being able to classify strongly regular graphs under certain parameters has not be lost on researchers the last 50 years. For instance, under what parameters do we have a strongly regular Cayley graph? The generating set for such a graph is called a partial difference set. It is known that there exist partial difference sets for groups of exponent 4 or less. In [6], Jim Davis and John Polhill describe how they are able to construct difference sets in groups C_2^r and C_4^r for all $r \geq 2$, which can then be used to construct partial difference sets. They also prove a new product construction for such sets. Through this method, they are able to show that there do not exist partial difference sets in groups of exponent 16. However, a question raised by Davis and Polhill is whether or not partial difference sets exist in groups of exponent 8. In particular, do they exist in the group $C_8 \times C_8$?

In this paper, the study of strongly regular graphs and that of Cayley graphs is combined to find all partial difference sets given specific sets of parameters. In particular, Cayley graphs on finite abelian groups G are sought. This allows for the use of character theory of the groups and rational idempotents in the group ring $\mathbb{C}[G]$ to simplify computations. Given parameter sets, partial difference sets are found through a collapsing process on our unknown generating set S of the Cayley graph, working with smaller cases to build sets S that yield strongly regular Cayley graphs which are thus partial difference sets. Through this method, we answer the existence question posed by Davis and Polhill. The significance of being able to classify strongly regular graphs under certain parameters has not be lost on researchers the last 50 years. For instance, under what parameters do we have a strongly regular Cayley graph? The generating set for such a graph is called a partial difference set. It is known that there exist partial difference sets for groups of exponent 4 or less. In [6], Jim Davis and John Polhill describe how they are able to construct difference sets in groups C_2^r and C_4^r for all $r \geq 2$, which can then be used to construct partial difference sets. They also prove a new product construction for such sets. Through this method, they are able to show that there do not exist partial difference sets in groups of exponent 16. However, a question raised by Davis and Polhill is whether or not partial difference sets exist in groups of exponent 8. In particular, do they exist in the group $C_8 \times C_8$?

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4.2 Second Section

The significance of being able to classify strongly regular graphs under certain parameters has not be lost on researchers the last 50 years. For instance, under what parameters do we have a strongly regular Cayley graph? The generating set for such a graph is called a partial difference set. It is known that there exist partial difference sets for groups of exponent 4 or less. In [6], Jim Davis and John Polhill describe how they are able to construct difference sets in groups C_2^r and C_4^r for all $r \geq 2$, which can then be used to construct partial difference sets. They also prove a new product construction for such sets. Through this method, they are able to show that there do not exist partial difference sets in groups of exponent 16. However, a question raised by Davis and Polhill is whether or not partial difference sets exist in groups of exponent 8. In particular, do they exist in the group $C_8 \times C_8$? The significance of being able to classify strongly regular graphs under certain parameters has not be lost on researchers the last 50 years. For instance, under what parameters do we have a strongly regular Cayley graph? The generating set for such a graph is called a partial difference set. It is known that there exist partial difference sets for groups of exponent 4 or less. In [6], Jim Davis and John Polhill describe how they are able to construct difference sets in groups C_2^r and C_4^r for all $r \geq 2$, which can then be used to construct partial difference sets. They also prove a new product construction for such sets. Through this method, they are able to show that there do not exist partial difference sets in groups of exponent 16. However, a question raised by Davis and Polhill is whether or not partial difference sets exist in groups of exponent 8. In particular, do they exist in the group $C_8 \times C_8$?

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It turns out that such Cayley graphs with our chosen set of parameters are negative Latin Square graphs. To discuss the significance of this, in Section II we begin by reviewing Cayley graphs and strongly regular graphs. This section is meant as a brief overview of the specific topics we need to utilize in this paper. Further study of these can be found in [8] and [2].

Section III describes how we use the rational idempotents in $\mathbb{C}[G]$ to find partial difference sets in G, throughout which we discuss the dual group G^* and properties of the characters in G^*. An example of this method is thoroughly investigated in Section IV, where we consider the group $C_8 \times C_8$ and parameter set $(64,18,2,6)$. From this construction we can now generate all partial differences sets in groups of exponent 8. Our method can also be extended to groups $C_{2^n} \times C_{2^n}$ with parameter set $(64,9r,r^2+3r-8,r^2+r)$ for positive integer values of r. We discuss the more general case and further topics of investigation in Section V.

Filtering is one of the most popular methods in image restoration. The local filtering restores an image by its local average such that the restored value at a pixel is obtained as a (weighted) average of its neighboring pixels. People show favor towards local filters for they provide fast and real-time algorithms. The local linear filtering algorithms, such as the well-known Gaussian filtering and Wiener filtering, are fast but tend to blur image. To avoid the blurring effect, many nonlinear filters have been constructed, for example, the Yaroslavsky neighborhood filter [18] (or sigma filter [10]) and the bilateral filter [17] (or SUSAN filter [16]). These filters, whose weights favor the pixels having the similar gray level values, create faster diffusion inside a homogeneous region while slower diffusion across the boundary of the region so that the image edge can be preserved. Although nonlinear filtering algorithms do not take the benefit of linear convolution, they can still be implemented very fast using very little read-only memory (ROM). Besides, being easily integrated into hardware, they are attracting more and more industrial attentions (see [4, 7, 12, 13, 19]).

Unfortunately, all of these local nonlinear filters create artificial shocks, showing a staircase effect. That is, artificial boundaries are created in the image of flat region. The authors of [3] showed that the Yaroslavsky neighborhood filter asymptotically behaves like a Perona-Malik model [14], creating large flat zones and spurious contours inside smooth regions. They also illustrated that this enhancing character of the neighborhood filter is due to its failure in reproducing linear functions (see Section IV in [3]). The similar behavior of the bilateral filter was also observed by Barash in [1] and Chui and Wang in [5].

To eliminate the staircase effect caused by the above nonlinear filters, the authors of [3] applied linear regression to a filter and called the corrected one linear regression neighborhood filter (LRNF), which is a sequel of a nonlinear filtering and its linear regression. Thanks to linear regression correction, LRNF no longer has enhancing character. However, the correction needs much more computing time, making LRNF slower than a standard filtering algorithm.

The purpose of this paper is to construct a new local nonlinear filter, called directional diffusion filter (DDF), which still preserves image edge, but does not show staircase effect. Because DDF reproduces linear functions, it has no need of linear regression correction and, therefore, is much faster than LRNF.

The paper is organized as follows. In the next section, we briefly review the linear regression correction method and analyze its necessity for the Yaroslavsky filter and the bilateral filter in eliminating staircase effect. In Section 3, we introduce the directional diffusion filter, reveal the relation between it and the improved TV model [11], and explore its ability to reproduce linear functions. In Section 4, we develop its numerical algorithm and discuss how to set the parameters in the filter. In the last section, we implement the algorithm in several experiments.

4.3 Review Of Linear Regression Correction For Nonlinear Filters

Let $u(x), x \in \Omega \subset \mathbb{R}^2$, be an image. As usual, we assume $\Omega = [0,1]^2$. The Yaroslavsky neighborhood filter [18] first defines a neighborhood $N(x) := G_h(x) \cap B_\rho(x)$ for pixel x, where $G_h(x) = \{y \in \Omega; \quad |u(y) - u(x)| < h\}$ and $B_\rho(x) = \{y \in \Omega; \quad ||y - x|| < \rho\}$, then sets the updated value at x as the average of the pixels in $N(x)$:

$$YNF_{h,\rho}u(x) = \frac{1}{|N(x)|} \int_{N(x)} u(y)dy,$$

where $|N(x)|$ denotes the Lebegues measure of $N(x)$. This filter is often rewritten in a more continuous form as

$$YNF_{h,\rho}u(x) = \frac{1}{C(x)} \int_{B_\rho(x)} u(y) e^{-\frac{|u(y)-u(x)|^2}{h^2}} dy,$$

where $C(x) = \int_{B_\rho(x)} e^{-\frac{|u(y)-u(x)|^2}{h^2}} dy$ is the normalization factor. If we also smooth the Yaroslavsky filter in the spatial domain, then it becomes the bilateral filter [17] (or SUSAN filter [16]):

$$BNF_{h,\rho}u(x) = \frac{1}{C(x)} \int_{\Omega} u(y) e^{-\frac{c\|x-y\|^2}{\rho^2} - \frac{|u(y)-u(x)|^2}{h^2}} dy, \tag{5}$$

where $C(x)$ is again the normalization factor in the form as

$$C(x) = \int_{\Omega} e^{-\frac{c\|x-y\|^2}{\rho^2} - \frac{|u(y)-u(x)|^2}{h^2}} dy.$$

Note that if we choose c such that $e^{-c} \approx 0$, then

$$BNF_{h,\rho} u(x) \approx \frac{1}{C(x)} \int_{B_\rho(x)} u(y) e^{-\frac{c\|x-y\|^2}{\rho^2} - \frac{|u(y)-u(x)|^2}{h^2}} dy.$$

Hence, no essential difference exists between the bilateral filter and the Yaroslavsky one.

CHAPTER 5

STRONGLY REGULAR CAYLEY GRAPHS

5.1 Assuptions

We assume throughout this section that G is a finite group and S is subset of G. The **Cayley graph** with respect to G is the graph $\Gamma = \Gamma(G,S)$ such that the vertex set V is exactly the members of G and, for all vertices $x, y \in V$, x is adjacent to y if and only if $y = sx$ for some $s \in S$.

For this paper, we consider only **simple** Cayley graphs. To achieve this, we choose our set S so that the identity element of G is not in S and if $s \in S$ then $s^{-1} \in S$. In addition to being simple, based on the group structure built into a Cayley graph, we note that our graphs are all regular.

A special type of regular graph, called strongly regular, was first introduced by R.C. Bose in 1963 [2]. Given a regular graph with v vertices of degree k, we can define a **strongly regular graph**, denoted $SRG(v,k,\lambda,\mu)$, to be such that there exist integers λ and μ satisfying that every pair of adjacent vertices have λ common neighbors and every pair of non-adjacent vertices have μ common neighbors. Probably the most famous example of a strongly regular graph is the Petersen graph, which is a $SRG(10,3,0,1)$:

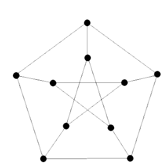

Figure 4: Petersen Graph

We can see here that the Petersen graph is a connected graph which satisfies the definition of a strongly regular graph, where adjacent vertices do not have common neighbors and non-adjacent vertices have one common neighbor. Let's recall that the Petersen graph is the complement of the line graph of the complete graph on five vertices, which is connected. For connected strongly regular graphs whose complements are also connected, we have the following bounds on their parameters:

$$0 < \mu < k < v - 1$$

Considering further the complement of a strongly regular graph, if we have a graph $\mathscr{G} = SRG(v,k,\lambda,\mu)$, then its complement is $\overline{\mathscr{G}} = SRG(v, v-k-1, v-2k+\mu-2, v-2k+\lambda)$. Because our parameters are non-negative, this tells us that

$$v - 2k + \mu - 2 \geq 0$$

5.2 Lemma

We can more carefully consider the structure of strongly regular graphs to develop the next lemma which specifies under what parameters we may construct a strongly regular graph.

Lemma 12. *For a strongly regular graph $\mathscr{G} = SRG(v,k,\lambda,\mu)$,*

$$k(k - \lambda - 1) = \mu(v - k - 1)$$

Proof. Fix a vertex x in $\mathscr{G}$, and consider the set $E := \{(y,z) : y \; z, \; x \; y \; x \nsim z\}$. We can computer the order of E in two different ways.

First, by counting the number of vertices of $\mathscr{G}$ which are distance 2 from x. Because the degree of x is k and the diameter of $\mathscr{G}$ is 2, there are $v-k-1$ vertices distance 2 from x. Note that μ counts the number of common neighbors of non-adjacent vertices. Thus,

$$|E| = \mu(v-k-1)$$

Now, let's determine the order of E by first counting the number of vertices of $\mathscr{G}$ which are adjacent to x not having a common neighbor with x. Recall that λ counts the number of common neighbors of adjacent vertices. Thus, there are $k-\lambda-1$ vertices of $\mathscr{G}$ adjacent to x and not adjacent to a neighbor of x. Because the degree of $\mathscr{G}$ is k,

$$|E| = k(k-\lambda-1).$$

$\square$

CHAPTER 6

FOR DEMO FIGS AND TABLES ONLY

6.1 Some Graphs

To add graphs in your thesis, you may use the following methods. The first figure is showing in the Chapter 2. The second figure is in pdf-format. You may use "scale" option to rescale it.

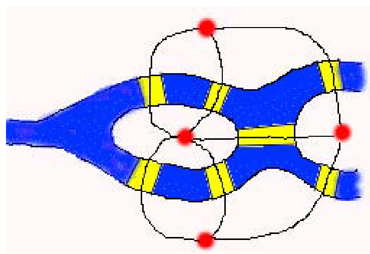

Figure 5: Seven Briges.

The third figure contains three graphs: Swiss Roll, S-Curve, and 3D Cluster, defined as follows.

- Parametric equation of the Swiss Roll:

$$\begin{cases} x = & (\frac{3\pi}{2}(1+2t))\cos(\frac{3\pi}{2}(1+2t)), \\ y = & s, \\ z = & (\frac{3\pi}{2}(1+2t))\sin(\frac{3\pi}{2}(1+2t)), \end{cases} \qquad 0 \le s \le L,\ |t| \le 1.$$

- Parametric equation of the S-Curve:

$$\begin{cases} x = & -\cos(1.5\pi t), \\ y = & s, \\ z = & \begin{cases} -\sin(1.5\pi t) & 0 \le t \le 1, \\ 2+\sin(1.5\pi t) & 1 < t \le 2, \end{cases} \end{cases} \qquad 0 \le s \le L,\ 0 \le t \le 2\ .$$

- Construction of the 3D Clusters: The 3D cluster in this experiment is not a surface in the usual sense, but consists of three separated balls, with centers connected by two line segments.

We show their graphs in the Figure 6.

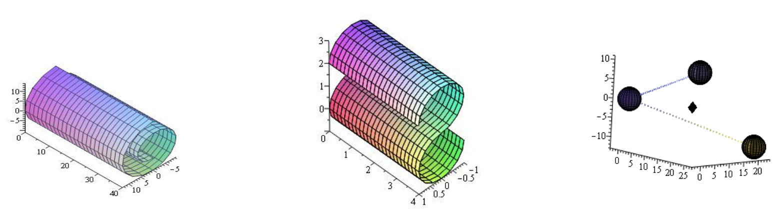

Figure 6: Left: Swiss Roll. Middle: S Curve. Right: 3D Cluster.

6.2 Some Tables

In the first example of the Swiss Roll dataset, Gaussian random projection (i.e. type-1) is used for RAT 1, $\cdots$, RAT 4, and the experimental DR results are compiled in Table 1. Observe that all RAT algorithms are much more efficient than the standard Isomap algorithm, while the deviations of each RAT from Isomap are negligible.

We assume throughout this section that G is a finite group and S is subset of G. The **Cayley graph** with respect to G is the graph $\Gamma = \Gamma(G,S)$ such that the vertex set V is exactly the members of G and, for all vertices $x, y \in V$, x is adjacent to y if and only if $y = sx$ for some $s \in S$.

For this paper, we consider only **simple** Cayley graphs. To achieve this, we choose our set S so that the identity element of G is not in S and if $s \in S$ then $s^{-1} \in S$. In addition to being simple, based on the group structure built into a Cayley graph, we note that our graphs are all regular.

A special type of regular graph, called strongly regular, was first introduced by R.C. Bose in 1963 [2]. Given a regular graph with v vertices of degree k, we can define a **strongly regular graph**, denoted $SRG(v,k,\lambda,\mu)$, to be such that there exist integers λ and μ satisfying that every pair of adjacent vertices have λ common neighbors and every pair of non-adjacent vertices have μ common neighbors. Probably the most famous example of a strongly regular graph is the Petersen graph, which is a $SRG(10,3,0,1)$:

Algorithm	CPU time	eigen 1	eigen 2	eigen 3	deviation
Isomaps	3.8179	1.0000	0.9543	0.0290	
RAT 1	0.0675	0.9972	0.9416	0.0173	0.0030
RAT 2	0.2253	1.0000	0.9542	0.0280	0.0001
RAT 3	0.5190	1.0000	0.9543	0.0285	0.0001
RAT 4	0.5916	1.0000	0.9543	0.0285	0.0000

Table 2: Comparison of standard Isomap algorithm with RAT applied to Isomap DRK: All RAT algorithms employ the 6-neighborhood and Gaussian (i.e. type-1) random projection. For normalization, all eigenvalues are divided by the first eigenvalue = 1654657 of the DRK.

In the second example of the S-curve dataset, Gaussian random projection (i.e. type-1) is again used for RAT 1, $\cdots$, RAT 4, and the experimental DR results are compiled in Table 2. Observe that all RAT algorithms are much more efficient than the standard Isomap algorithm, while the deviations of each RAT from Isomap are again negligible. We assume throughout this section that G is a finite group and S is subset of G. The **Cayley graph** with respect to G is the graph $\Gamma = \Gamma(G,S)$ such that the vertex set V is exactly the members of G and, for all vertices $x,y \in V$, x is adjacent to y if and only if $y = sx$ for some $s \in S$.

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In the third example of the 3D cluster dataset, Gaussian random projection (i.e. type-1) is again used for RAT 1, $\cdots$, RAT 4, and the experimental DR results are compiled in Table 3. Observe that all RAT algorithms are much more efficient than the standard Isomap algorithm, while the deviations of each RAT from Isomap are again negligible. We assume

Algorithm	CPU time	eigen 1	eigen 2	eigen 3	deviation
Isomaps	3.4289	1.0000	0.2762	0.0199	
RAT 1	0.0642	0.9999	0.2753	0.0099	0.0023
RAT 2	0.2043	1.0000	0.2762	0.0190	0.0001
RAT 3	0.5109	1.0000	0.2762	0.0197	0.0001
RAT 4	0.5662	1.0000	0.2762	0.0197	0.0001

Table 3: Comparison of standard Isomap algorithm with RAT applied to Isomap DRK: All RAT algorithms employ the 6-neighborhood and Gaussian (i.e. type-1) random projection. For normalization, all eigenvalues are divided by the first eigenvalue = 17553 of the DRK.

throughout this section that G is a finite group and S is subset of G. The **Cayley graph** with respect to G is the graph $\Gamma = \Gamma(G,S)$ such that the vertex set V is exactly the members of G and, for all vertices $x, y \in V$, x is adjacent to y if and only if $y = sx$ for some $s \in S$.

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Algorithm	CPU time	eigen 1	eigen 2	eigen 3	deviation
Isomaps	4.3294	1.0000	0.1454	0.0049	
RAT 1	0.0756	0.9999	0.1453	0.0038	0.0009
RAT 2	0.2123	1.0000	0.1454	0.0049	0.0000
RAT 3	0.7380	1.0000	0.1454	0.0049	0.0000
RAT 4	0.7103	1.0000	0.1454	0.0049	0.0000

Table 4: Comparison of standard Isomap algorithm with RAT applied to Isomap DRK: All RAT algorithms employ the 6-neighborhood and Gaussian (i.e. type-1) random projection. For normalization, all eigenvalues are divided by the first eigenvalue = 22271 of the DRK.

If your table is very wide, you can use the following method. The example below is gotten from Darwin Luna's thesis.

Table 5: Heat Diffusion 10 steps with source term

$\Delta t \diagdown \Delta x$	0.00000	0.10000	0.20000	0.30000	0.40000	0.50000	0.60000	0.70000	0.80000	0.90000	1.00000
0	.0.00000	0.30000	0.60000	0.90000	1.20000	1.50000	1.40000	1.30000	1.20000	1.10000	0.00000
.01	0.00000	0.29660	0.58980	0.87280	1.12860	1.31301	1.31042	1.21826	1.04435	0.71478	0.00000
.02	0.00000	0.28876	0.56967	0.83045	1.04889	1.18762	1.20096	1.10484	0.89529	0.53669	0.00000
.03	0.00000	0.27718	0.54279	0.78151	0.97130	1.08348	1.09154	0.99017	0.77413	0.43694	0.00000
.04	0.00000	0.26307	0.51203	0.73022	0.89712	0.98983	0.98890	0.88533	0.67692	0.37129	0.00000

REFERENCES

[1] D. Barash, *A fundamental relationship between bilateral filtering, adaptive smoothing, and the nonlinear diffusion equation*, IEEE Trans. Pattern Anal. Mach. Intell. **24** (2002), no. 6, 844–847.

[2] A. E. Brouwer, A. M. Cohen, and A. Neumaier, *Distance-regular graphs*, Ergebnisse der Mathematik und ihrer Grenzgebiete (3) [Results in Mathematics and Related Areas (3)], vol. 18, Springer-Verlag, Berlin, 1989. MR 1002568 (90e:05001)

[3] A. Buades, B. Coll, and J.-M. Morel, *The staircasing effect in neighborhood filters and its solution*, IEEE Transactions on Image Processing **15** (2006), no. 6, 1499–1505.

[4] C. K. Chui and J. Z. Wang, *Image clean-up and pre-coding*, 06 2005.

[5] C. K. Chui and J. Wang, *PDE models associated with the bilateral filter*, Adv. Comput. Math. **31** (2009), no. 1-3, 131–156. MR 2511577 (2010e:94028)

[6] J. A. Davis and J. Polhill, *Difference set constructions of DRADs and association schemes*, J. Combin. Theory Ser. A **117** (2010), no. 5, 598–605. MR 2600978 (2011h:05038)

[7] P. Devaney, D. Gnanaprakasam, and T. Leacock, *Post-filter for removing ringing artifacts of dct coding*, 10 1998.

[8] J.-M. Goethals and J. J. Seidel, *Strongly regular graphs derived from combinatorial designs*, Canad. J. Math. **22** (1970), 597–614. MR 0282872 (44 #106)

[9] J. P. S. Kung and G.-C. Rota, *The invariant theory of binary forms*, Bull. Amer. Math. Soc. (N.S.) **10** (1984), no. 1, 27–85. MR 722856 (85g:05002)

[10] J.-S. Lee, *Digital image smoothing and the sigma filter*, Computer Vision, Graphics, and Image Processing **24** (1983), no. 2, 255–269.

[11] A. Marquina and S. Osher, *Explicit algorithms for a new time dependent model based on level set motion for nonlinear deblurring and noise removal*, SIAM J. Sci. Comput. **22** (2000), no. 2, 387–405 (electronic). MR 1780606 (2002d:65078)

[12] R. Maurer, *Method for bilateral filtering of digital images*, 04 2008.

[13] R. Maurer and D. Barash, *Method for enhancing compressibility and visual quality of scanned document images*, 05 2004.

[14] P. Perona and J. Malik, *Scale-space and edge detection using anisotropic diffusion*, IEEE Trans. Pattern Anal. Mach. Intell. **12** (1990), no. 7, 629–639.

[15] P. Roquette, *Emmy Noether and Hermann Weyl*, Groups and analysis, London Math. Soc. Lecture Note Ser., vol. 354, Cambridge Univ. Press, Cambridge, 2008, pp. 285–326. MR 2528471

[16] S. M. Smith and J. M. Brady, *Susan–a new approach to low level image processing*, Int. J. Comput. Vision **23** (1997), no. 1, 45–78.

[17] C. Tomasi and R. Manduchi, *Bilateral filtering for gray and color images*, Proceedings of the Sixth International Conference on Computer Vision (Washington, DC, USA), ICCV '98, IEEE Computer Society, 1998, pp. 839–.

[18] L. P. Yaroslavsky, *Digital picture processing*, Springer Series in Information Sciences, vol. 9, Springer-Verlag, Berlin, 1985, An introduction. MR 818222 (88b:94001)

[19] M. Zhang and R. Mathew, *Method and decoder system for reducing quantization effect of a decoded image*, 10 2004.

APPENDIX A

Write your Appendix content here. (Sample)

The following style manuals have been accepted by the Sam Houston State University Graduate Council. The most recent edition of these manuals should always be followed. CONSULT YOUR THESIS DIRECTOR TO DETERMINE WHICH MANUAL IS REQUIRED BY YOUR DEPARTMENT.

ACS (American Chemical Society) Style Guide: A Manual for Authors and Editors

AIP (American Institute of Physics) Style Manual

Associated Press Stylebook and Libel Manual

Chicago Manual of Style

Form and Style: Theses, Reports, Term Papers (William G. Campbell)

A Manual for Authors of Mathematical Papers (American Mathematical Association)

A Manual for Writers of Term Papers, Theses, Dissertations (Kate Turabian)

MLA (Modern Languages of America) Handbook for Writers of Research Papers

Publication Manual of the American Psychological Association

Scientific Style and Format: THE CBE (Council of Biology Editors) Manual for Authors, Editors and Publishers

Style Manual for Political Science

Style Manual (United States Government Printing Office)

Suggestions to Authors of the Reports of the United States Geological Survey

A Uniform System of Citation (Harvard Law Review)

A Manual for Authors of Mathematical Papers (American Mathematical Association)

A Manual for Writers of Term Papers, Theses, Dissertations (Kate Turabian)

MLA (Modern Languages of America) Handbook for Writers of Research Papers

Publication Manual of the American Psychological Association

Scientific Style and Format: THE CBE (Council of Biology Editors) Manual for Authors, Editors and Publishers

Style Manual for Political Science

Style Manual (United States Government Printing Office)

Suggestions to Authors of the Reports of the United States Geological Survey

A Uniform System of Citation (Harvard Law Review)

APPENDIX B

Anomaly detection. Detecting anomaly for given statistical models.

Classification of objects (Spectral classification). Classifying objects in a HSI data set.

Demixing. Finding material components in a raster cell.

Electromagnetic radiation (EMR). The energy in the form of electromagnetic waves.

Electromagnetic spectrum. The entire family of electromagnetic radiation, together with all its various wavelengths.

Endmember spectra. The "pure" spectra that contribute to mixed spectra.

Fused images. A fused image is a combination of the HSI image and the HRI image (to be mentioned below). It is usually the best because of high resolution from the HRI camera and color information from the HSI sensor. This combination results in sufficiently high image resolution and contrast to facilitate image evaluation by the human eyes.

High Resolution Imagery (HRI). A high-resolution image (HRI) camera, which captures black-and-white or panchromatic images, is usually integrated in an HSI system to capture the same reflected light. However, the HRI camera does not have a diffraction grating to disperse the incoming reflected light. Instead, the incoming light is directed to a wider CCD (Charge-Couple Device) to capture more image data. The HSI resolution is typically one meter per pixel, and the HRI resolution is much finer: typically a few inches square per pixel.

Hyperspectral imaging. The imagery consists of a larger number of spectral bands so that the totality of these bands is numerically sufficient to represent a (continuous) spectral curve for each raster cell.

Illumination factors. The incoming solar energy varies greatly in wavelengths, peaking in the range of visible light. To convert spectral radiance to spectral reflectance, the illumination factors must be accounted. Illumination factors taken into account includes both the illumination geometry (angles of incoming light, etc.,) and shadowing. Other factors, such as atmospheric and sensor effects, are also taken into consideration.

Macroscopic and intimate mixtures. Microscopic mixture is a linear combination of its endmembers, while an intimate mixture is a nonlinear mixture of its endmembers.

Mixed spectra. Mixed spectra, also known as composite spectra, are contributed by more than one material components.

Multi-spectral imaging. The imaging bins the spectrum into a handful of bands.

Raster cell. A pixel in a hyperspectral image.

Reflectance conversion. Radiance values must be converted to reflectance values before comparing image spectra with reference reflectance spectra. This is called atmospheric correction. The method for converting the radiance to reflectance is also called reflectance conversion. The image-based correction methods include *Flat Field Conversion* and *Internal Average Relative Reflectance Conversion*. They apply the model $R_1 = mR_2$, where R_1 is the reflectance, R_2 is the radiance, and m is the conversion slope. Some conversions also apply the linear model $R_1 = -c + mR_2$, where c is an offset that needs to be abstracted from the radiance. The popular conversions are

1. *Flat Field Conversion*. A flat field has a relatively flat spectral reflectance curve. The mean spectrum of such an area would be dominated by the combined effects of solar irradiance and atmospheric scattering and absorption. The scene is converted to "relative" reflectance by dividing each image spectrum by the flat field mean spectrum.

2. *Internal Average Relative Reflectance (IARR) Conversion*. This technique is used when no knowledge of the surface materials is available. The technique calculates a relative reflectance by dividing each spectrum (pixel) by the scene average spectrum.

Region segmentation. Partitioning the spatial region of a hyperspectral image into multiple regions (sets of pixels). The goal of segmentation is to simplify and/or change the representation of an HSI image into something that is more meaningful and easier to analyze. Region segmentation is typically used to locate objects and boundaries (lines, curves, etc.) in images.

Remote sensing. Sensing something from a distance. The following processes affect the light that is sensed by a remote sensing system:

1. *Illumination*. Light has to illuminate the ground and objects on the ground before they can reflect any light. In the typical remote sensing environment, which is outdoors, illumination comes from the sun. We call it solar illumination.

2. *Atmospheric Absorption and Scattering of Illumination Light*. As solar illumination travels through the atmosphere, some wavelengths are absorbed and some are scattered. Scattering is the change in direction of a light wave that occurs when it strikes a molecule or particle in the atmosphere.

3. *Reflection*. Some of the light that illuminates the ground and objects on the ground is reflected. The wavelengths that are reflected depend on the wavelength content of the illumination and on the object's reflectance. The area surrounding a reflecting object also reflects light, and some of this light is reflected into the remote sensor.

4. *Atmospheric Absorption and Scattering of Reflected Light*. As reflected light travels through the atmosphere to the remote sensor, some wavelengths are absorbed, some are scattered away from the sensor, and some are scattered into the sensor.

These four effects all change the light that reaches the remote sensor from the original light source. After the reflected light is captured by the remote sensor, the light is further affected by how the sensor converts the captured light into electrical signals. These effects that occur in the sensor are called sensor effects.

Spectral curve. It is the one-dimensional curve of a spectral reflectance.

Spectral libraries. A spectral library consists of a list of spectral curves with data of their characteristics corresponding to specific materials such as mines, plants, etc.

Spectral radiance. It is the measurable reflected light reaching the sensor. The spectral reflectance of the material is only one factor affecting it. It is also dependent of the spectra of the input solar energy interactions of this energy during its downward and upward passages through the atmosphere, etc

Shape recognition. Recognizing the shape of a detected object.

Spectral reflectance. It is the ratio of reflected energy to incident energy as a function of wavelengths. A certain material has its own spectral reflectance. The light that is reflected by an object depends on two things: (1) light that illuminates the object; and (2) the reflectance of the object. Reflectance is a physical property of the object surface. It is the percentage of incident EMR of each wavelength that is reflected by the object. Because it is a physical property, it is not affected by the light that illuminates the object.

Spectral reflection. It is the observed reflected energy, represented as a function of wavelengths under illumination. It is affected by both reflectance of the object and the light that illustrates the object. If an object was illuminated by balanced white light, and if there was no atmospheric absorption or scatter, and if the sensor was perfect, then the wavelength composition of reflected light detected by an HSI sensor would match the reflectance, or spectral signature of the object.

Spectral space. The *n*-dimensional space, where each point is the spectra of a material or a group of materials.

Spectral signature. A unique characteristic of an object, represented by some chart of the plot of the object's reflectance as a function of its wavelength. It can be thought of as an EMR "fingerprint" of the object.

Spectroscopy. The study of the wavelength composition of electromagnetic radiation. It is fundamental to how HSI technology works.

Signature matching. Matching reflected light of pixels to spectral signatures of given objects.

VITA

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EDUCATION

Master of Science student in Mathematics at Sam Houston State University, August 2010 – present. Thesis title: "On constructing form systems for ternary quartic forms: A translation and modern interpretation of Emmy Noether's doctoral dissertation."

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ACADEMIC EMPLOYMENT

Graduate Teaching Assistant, Department of Mathematics and Statistics, Sam Houston State University, August 2010 - present. Responsible of delivering every lecture, and all other aspects of course management, such as, creating activities to monitor learning, grading and recording grades, suggesting solutions to assigned problems, addressing student performance issues, developing ways to improve learning and understanding, assigning final grades..

Research Assistant to Luís García-Puente, Department of Mathematics and Statistics, Sam Houston State University, Fall 2007 - May 2010. Learned technical material relevant to the research project, learned dedicated computer software, created software modules to create and manage a large database of computations related to the project, designed and managed a php website to display the computational results of the project, co-authored a peer-reviewed research publication.

PUBLICATIONS

Luís García-Puente, Sarah Spielvogel, and Seth Sullivant. 2010. Identifying Casual Effects with Computer Algebra. Proceedings of the 26th Conference of Uncertainty in Artificial Intelligence.

PRESENTATIONS AT PROFESSIONAL MEETINGS

Student, Sarah Spielvogel, and Luís García-Puenter. Causality - something something. Texas Undergraduate Math Conference, Sam Houston State University, Huntsville, Texas, 24 June 2004.

Student, Sarah Spielvogel, and Luís García-Puenter. Causality - something something. SACNAS Conference, San , California, 24 June 2004.

Student, Sarah Spielvogel, and Luís García-Puenter. Causality - something something. Joint Math Meetings, San Francisco, California, 24 June 2004.

ACADEMIC AWARDS

Outstanding Transfer Student, Department of Mathematics and Statistics, Sam Houston State University, April 2009.

Recipient of Glenda St Andrie Scholarship 2009-2010

Member of Pi Mu Epsilon

Deans List Fall 2007, 2008