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Product Upgrade and Time-to-Market

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Abstract

The introduction of upgraded products is challenging, especially in a market consisting of two consumer segments with different valuations for quality. In this paper, we apply the self-selection approach to study optimal product upgrading strategies. We propose a two-period product-upgrade model with the time-to-market of upgraded products incorporated. The model profit is benchmarked with that of a one-period model without product upgrading. We identify the conditions that are optimal to introduce upgraded products. The degree of cannibalization and the time-to-market are shown to be decisive factors. In particular, the product-upgrade model is optimal when the degree of cannibalization is small and when the time-to-market of upgraded products is less than a threshold value. Otherwise, the one-period benchmark model is optimal.

1. Introduction

Firms periodically introduce upgraded products. Microsoft released its popular operating system Windows XP in October of 2001, an upgraded Windows Vista in November of 2006 and the most recent, more user-friendly Windows 7 in November of 2009. The prominent Apple product iPhone was first launched in June 29, 2007. Its next version iPhone 3G followed one year later and was on the market on July 11, 2008.

The introduction of upgraded products is challenging, especially in a market consisting of two consumer segments with different valuations for quality. Market segmentation provides firms opportunities to offer differentiated products to segmented consumers. If firms can efficiently direct the differentiated products to their intended consumer segments, i.e., low-segment consumers purchase a basic-quality product and high-segment consumers purchase an upgraded-quality product, then higher profits could be obtained.
However, in a segmented market, the demand of upgraded products could be cannibalized. Upgraded products are usually targeted to consumers in the high-segment, which is the segment with a higher valuation. The other segment with a lower valuation is defined as low-segment. Under certain product configurations, high-segment consumers may purchase a product that is intended for low-segment consumers. Profit will be less under this cannibalization situation. It may be optimal for firms not to offer upgraded products if cannibalization becomes too serious.

In addition, the time-to-market of upgraded products has a great impact on consumer’s purchasing decisions. High-segment consumers need to wait a certain period of time for upgraded products. The longer the waiting time for upgraded products, the lower the product valuations would be for consumers. What’s more, the longer the introduction time of upgraded products, the less the present value of profit would be for firms.

Firms need to take into consideration the issues we discussed above in the decisions of upgraded products introduction. Particularly, many important questions need to be addressed. How would the degree of cannibalization affect the introduction of upgraded products? When is it optimal to offer (or not to offer) upgraded products? What would be the optimal product configurations when there is product upgrading and when there is not? What would be the optimal product introduction strategy with or without product upgrading? We develop a self-selection model in this paper to answer these questions.

We first consider a one-period benchmark model with no product upgrading, and conclude the optimal product introduction strategies. We then present a product-upgrade model, where a basic-quality product is offered first and its upgraded version one period after. The time-to-market of the upgraded product is incorporated into the discount factors of valuation and profit in the second period. We characterize the optimal product upgrading strategies, after comparing the profits from the two models. In particular, we identify the conditions that are optimal to introduce the upgraded product. We find that the product-upgrade model is optimal when the degree of cannibalization is small and when the time-to-market of upgraded products is less than a threshold value. Otherwise, the one-period benchmark model is optimal.

The remainder of this paper is organized as follows. We review the literature in the next section. Model setups are described in Section 3. A benchmark model and its results are presented in Section 4. The product-upgrade model and its result analysis can be found in
Section 5. In the last section, we summarize the paper and remark on future directions. All proofs are deferred to the Appendix for clarity of presentation.

2. Literature Review


Our model is mostly related to Moorthy and Png (1992) and Mallik and Chhajed (2002). In their works, both low-end and high-end products are available in the two considered periods. The decision to make, is which product(s) to offer in which period. By contrast, we consider a product upgrading scenario, where the upgraded product, if introduced, is only available in the second period. The decision to make, is whether to introduce the upgraded product or not.

What's more, time-to-market is another focus in our model. We incorporate the time-to-market of the upgraded product in model formulation, and provide timing conditions that are optimal for product upgrading. Cohen et al. (1996) provide a good example in the time-to-market analysis. They model the tradeoff between time to market and product performance. We focus on the time-to-market's negative impacts on valuation and profit in the second period.

3. Model Setups
In this section, we describe consumer and product configurations, as well as model assumptions and setups.

3.1 Products

Consider a product that may be differentiated by some attributes, for example, profile and battery life for cell phones, security and software compatibility in operating systems, style and fabric for clothes, etc. We call this attribute “quality” and denote it by $q$. We consider a two-period model. A monopolist produces and sells a basic-quality product in the first period. Its quality level is denoted by $q_L$. The first period lasts time $t$, where $t \geq 0$, during which the quality of the product will be improved and upgraded. The upgraded product with a higher quality $q_H$ will be offered in the second period. We assume only one set of product configurations (price and quality) is offered in each period, due to resource constraints.

We further assume the product is a durable product. Durable goods provide a significant service period to consumers and are often considered a one-time buy. Thus, a consumer will leave the market forever, once she has bought a unit of the product, regardless of quality. In other words, we do not consider repeated purchase.

The cost of manufacturing the product is assumed to be quadratic in the quality level with coefficient $c$. This assumption is consistent with the literature (Moorthy and Png, 1992, Kim and Chhajed, 2002, and Mallik and Chhajed, 2006). Specifically, for quality level $q$, the manufacturing cost is $cq^2$.

3.2 Consumers

Consider a market consisting of two segments: a high segment $H$ and a low segment $L$. The size of segment $H$ is $n_H$ and the size of segment $L$ is $n_L$. The high segment values quality level $q$ at $\theta_H q$, and the low segment values it at $\theta_L q$, where $\theta_H > \theta_L$. Here $\theta_i (i = H, L)$ is the marginal valuation of the consumer per unit of quality. The term $\theta_i q$ is the consumer’s maximum willingness to pay for a quality level $q$. The utility $U_i$ that a consumer, in segment $i (i = H, L)$, derives from a product is $U_i = \theta_i q - p$, where $p$ is the price of the product. The consumer only purchases the product that gives her maximum non-negative utility. If no product gives her a positive utility, she can choose not to buy any product.
We assume consumer's marginal valuation \( \theta_i \) decreases in time. The longer a consumer has to wait to consume a product, the less desirable the product would be. At the end of a waiting time \( t \), we assume a consumer's marginal valuation diminishes to \( \theta_i e^{-t} \). Particularly, when \( t = 0 \), we obtain \( \theta_i e^{-t} = \theta_i \). Consumer's marginal valuation does not change when there is no waiting. When \( t = \infty \), we obtain \( \theta_i e^{-t} = 0 \). If the waiting time for the product is infinite, then the product has no value for the consumer anymore.

### 3.3 Model Objectives

The monopolist's objective is to maximize the profit (the product price minus manufacturing cost and multiplied by the number of consumers in each segment). The monopolist sells the basic-quality product in the first period and only offers the upgraded-quality product in the second period. The profit obtained from the second period will be discounted by a time sensitive factor \( e^{-t} \), which is decreasing in time \( t \). Again, when \( t = 0 \), the time sensitive factor becomes \( e^{-t} = 1 \). The present value of the profit stays the same when there is no gap between the first and the second period. When \( t = \infty \), the factor is \( e^{-t} = 0 \). If the second period profit can only be realized in the infinite future, then its present value would be zero.

For maximum profit, the monopolist would like to target the basic-quality product to the L segment and the upgraded-quality product to the H segment. However, the consumers can self-select the two types of products. They may buy either of them, but not necessarily the one targeted at their segment. Therefore, the monopolist has to decide the product qualities and prices in such a way that it would be optimal for the consumers to choose the products targeted at their segment. The consumer's objective is to maximize the utility in their purchasing decision.

We further assume the monopolist commits in advance to the timing and product configurations. Therefore, H segment could evaluate the upgraded product accordingly and decide whether to wait and buy in the second period.

### 4. The Benchmark Model

We first consider a benchmark model, where the monopolist offers a product and there is no product upgrading afterwards. The product quality is \( q \) and its price is \( p \). The product utility is therefore \( U_L = \theta_L q - p \) for L segment and \( U_H = \theta_H q - p \) for H segment. Note that \( U_H > U_L \) since \( \theta_H > \theta_L \). Recall that the consumer only purchases if the product gives her non-negative
utility. Consequently, the model has two possible scenarios, in which either both segments buy the product or only H segment buys.

**Scenario I: both H and L segments buy**

In this scenario, \( U_L \geq 0 \) and \( U_H \geq 0 \), and both H and L segments buy the product. The monopolist's profit maximization problem is given below:

\[
\text{Max } \pi = (n_L + n_H) \cdot (p - cq^2)
\]  

(1)

In this scenario, the monopolist would extract the entire utility of L segment by letting \( U_L = 0 \) and setting product price \( p = \theta_L q \). The utility of H segment is strictly positive. The monopolist cannot extract the entire utility of H segment. Otherwise, the L segment would not purchase because its utility would become negative.

Substitute \( p = \theta_L q \) into function (1), and maximize the monopolist's profit with respect to \( q \), we obtain the optimal solutions below:

\[
q^* = \frac{\theta_L}{2c}, \ p^* = \frac{\theta^2_L}{2c}, \ \text{and } \pi^* = \frac{(n_L + n_H)\theta^2_L}{4c}.
\]  

(2)

**Scenario II: only H segment buys**

In this scenario, \( U_H \geq 0 \) and \( U_L < 0 \), only H segment buys the product. Note that there is no scenario such that only L segment buys. The utility of H segment is always greater than that of L segment. Whenever L segment buys, H segment will buy because of a positive utility.

The monopolist's problem becomes:

\[
\text{Max } \pi = n_H \cdot (p - cq^2)
\]  

(3)

The monopolist would extract the entire utility of H segment in this scenario. The product price is therefore \( p = \theta_H q \). Substitute the price into function (3), and maximize with respect to \( q \). The optimal solutions of the problem are:

\[
q^* = \frac{\theta_H}{2c}, \ p^* = \frac{\theta^2_H}{2c}, \ \text{and } \pi^* = \frac{n_H \theta^2_H}{4c}.
\]  

(4)

The two scenarios have different optimal configurations of the product. The quality and the price of the product are low in the first scenario. The product directly targets the L segment. H segment also buys because of a positive utility. In the second scenario, the product configurations, with a better quality and a higher price, are designed to attract H segment only.
Recall that the monopolist can offer only one set of product configurations due to resource constraints. Which set of product configurations to offer and which consumer segment to target? By comparing the optimal profits in the two scenarios, we conclude the following proposition:

**Proposition 1:** If \( R \leq \frac{\theta_L}{\theta_H + \theta_L} \), where \( R = \frac{n_H}{n_L} \left( \frac{\theta_H - \theta_L}{\theta_L} \right) \), it is optimal for the monopolist to offer a low quality product to both \( H \) and \( L \) segments. If \( R > \frac{\theta_L}{\theta_H + \theta_L} \), it is optimal for the monopolist to offer a high quality product to attract \( H \) segment only.

The term \( R = \frac{n_H}{n_L} \left( \frac{\theta_H - \theta_L}{\theta_L} \right) \) is a measurement of potential cannibalization as used in Moorthy and Png (1992) and Mallik and Chhajed (2006). The larger the value of \( R \), the more serious the cannibalization problem would be. According to Proposition 1, when the degree of cannibalization is serious \( (R > \frac{\theta_L}{\theta_H + \theta_L}) \), the monopolist is better off to sell to only \( H \) segment, instead of to both segments, by offering a high quality product.

5. The Product-Upgrade Model

In the product-upgrade model, the monopolist first offers a basic-quality product targeted to \( L \) segment. Product improvement then takes place afterwards, and an upgraded-quality product targeted to \( H \) segment is introduced after a period of time. The model sequence is illustrated in Figure 1, where \( L \) segment chooses the basic-quality product (with quality \( q_L \) and price \( p_L \)) in the first period, and \( H \) segment buys the upgraded-quality product (with quality \( q_H \) and price \( p_H \)) in the second period. The waiting time between the two periods is \( t \).

![FIGURE 1: The Product-Upgrade Model](image)

**Consumers:**
- \( L \) Segment: \( n_L \)
- \( H \) Segment: \( n_H \)

**Product:**
- Basic-quality \((q_L, p_L)\)
- Upgraded-quality \((q_H, p_H)\)
The monopolist’s profit maximization problem is described below.

\[
\begin{align*}
\text{Max} \quad & \pi = n_L \cdot (p_L - c q_L^L) + e^{-t} \cdot n_H \cdot (p_H - c q_H^H) \\
\text{subject to} \quad & \theta_L q_L - p_L \geq e^{-t} \theta_L q_H - p_H \\
& e^{-t} \theta_H q_H - p_H \geq \theta_H q_L - p_L \\
& \theta_L q_L - p_L \geq 0 \\
& e^{-t} \theta_H q_H - p_H \geq 0
\end{align*}
\] (5)

The objective function (5) is the total discounted profit for the monopolist. The first part represents the profit obtained from selling the basic-quality product in period one. The second part is a discounted profit from period two, where \(e^{-t}\) is the discount factor.

The constraint (6) is the self-selection constraint for L segment. The left side of the constraint is the utility of L segment when it buys the basic-quality product in period one. The right side of the constraint is the utility of L segment when it chooses the upgraded-quality product in period two. Note that L segment’s marginal valuation becomes \(\theta_L e^{-t}\) in period two because of the waiting time \(t\). The constraint (6) ensures that L segment buys the basic-quality product in period one, because of a higher utility. Similarly, the constraint (7) is the self-selection constraint for H segment. It directs H segment to choose the upgraded-quality product in period two.

The constraints (8) and (9) are the participation constraints for L segment and H segment respectively. The two constraints guarantee each segment will obtain a non-negative utility from purchasing the products, rather than not buy anything at all.

If the monopolist tries to extract the entire utility from the H segment by setting \(e^{-t} \theta_H q_H = p_H\), then these consumers will switch to the basic-quality product. By doing so, they will get a positive utility, because \(\theta_H q_L - p_L > \theta_L q_L - p_L \geq 0\). However, nothing will prevent the monopolist from extracting the L segment’s entire utility. The monopolist should choose price \(p_L\) so that constraint (8) binds:

\[
p_L = \theta_L q_L
\] (10)

The self-selection constraints (6) and (7) cannot be both binding because of our assumption \(\theta_H > \theta_L\). H segment is more willing to pay a higher price for additional quality. The monopolist should direct the upgraded-quality product to them and set price \(p_H\) so that constraint (7) binds.
By doing so, the monopolist is able to charge a higher price $p_H$. Consequently, the monopolist obtains a higher profit. It is easy to verify that the binding constraint (7) yields a higher value of $p_H$ than the binding constraint (6). The binding constraint (7) indicates H segment will obtain the same amount of utility from the two types of products. Even H segment is indifferent between the two products, the consumers will choose the upgraded-quality product for its better quality.

We obtain the expression of $p_H$ from the binding constraint (7):

$$p_H = e^{-t} \theta_H q_H - \theta_H q_L + p_L$$

(11)

Substitute (10) into (11), we obtain the value of $p_H$ in terms of $q_H$ and $q_L$:

$$p_H = e^{-t} \theta_H q_H - \theta_H q_L + \theta_L q_L$$

(12)

Substitute (10) and (12) into the monopolist’s profit function (5), and maximize with respective to $q_H$ and $q_L$, we obtain the optimal solutions of the model:

$$q_H^* = \frac{e^{-t} \theta_H}{2c}, \quad q_L^* = \frac{\theta_L}{2c} (1 - e^{-t} R)$$

(13)

$$p_H^* = \frac{e^{-t} \theta_H}{2c} - \frac{\theta_L n_R}{2cn_H} (1 - e^{-t} R); \quad p_L^* = \frac{\theta_L}{2c} (1 - e^{-t} R)$$

(14)

$$\pi^* = \frac{e^{-t} \theta_H \theta_L}{4c} + \frac{n_H \theta_L^2}{4c} (1 - e^{-t} R)^2$$

(15)

We next analyze the model results, present interesting findings and provide managerial insights.

Note that the value of the upgraded-quality should be higher than that of the basic-quality. Also the value of the basic-quality should be positive. These let us conclude the following result immediately.

**Proposition 2:** For the optimal solutions to be feasible, the degree of cannibalization needs to be within a certain range, specifically, $\max [0, e^t - \frac{\theta_H}{\theta_L}] < R < e^t$;

When $R \geq e^t$, the degree of cannibalization would be too serious to offer the upgraded product in the second period. Instead, the monopolist would choose not to upgrade the product. The upper limit of $R$ is obtained by requiring $q_L^*$ in (13) to be positive. The lower limit of $R$ guarantees the quality of the upgraded product is higher.
The most important questions to ask are whether and when it is profitable to offer the upgraded product. The answers can be obtained by comparing the optimal profit of the product-upgrade model with the ones in the benchmark model that has no product upgrading. We investigate the following two cases based on the optimality conditions in the benchmark model as concluded in Proposition 1.

**Case I:** $R \leq \frac{\theta_L}{\theta_H+\theta_L}$

First, we consider the situation when the degree of cannibalization is low, i.e., $R \leq \frac{\theta_L}{\theta_H+\theta_L}$, where L segment is targeted in the benchmark model. In this case, the monopolist chooses between two strategies: (i) target only L segment and offer a low quality product alone; (ii) target both segments and offer both a basic-quality product and its upgraded version one period after. Compare the profits of the two strategies in (15) and (2), we obtain the following proposition.

**Proposition 3:** In case I, it is optimal for the monopolist to offer the upgraded product if $R \leq \min \left( \frac{\theta_L}{\theta_H+\theta_L}, R_1 \right)$, where $R_1 = e^t - \sqrt{\frac{e^{2t(n_L+n_H)}\theta_H^2-e^{-t}n_H\theta_H^2}{n_L\theta_L^2}}$. Otherwise, it is optimal to offer the low quality product alone.

Proposition 3 demonstrates that product-upgrade is optimal only if the degree of cannibalization is relatively low, i.e., $R$ is smaller than $R_1$ and is within the feasible range of case I. Otherwise, the basic-quality product would cannibalize the demand for the upgraded-quality product, which would lower the profit of the product-upgrade model. When the degree of cannibalization becomes too serious in case I, the monopolist would be better off to target L segment only and to offer a low quality product alone.

Additionally, the value of $R_1$ needs to be positive for the monopolist to have an optimal region to offer the upgraded product. Otherwise, the optimal strategy for the monopolist is to target L segment in the first period and to offer no product upgrading afterwards. To demonstrate this, we assume $R_1$ to be negative. The condition in Proposition 3 becomes a negative value, which is not feasible. Consequently, it is not optimal to offer the upgraded product when $R_1$ is negative. Summarizing the above discussion, we obtain the following Corollary.
**Corollary 1:** In case I, under the feasible condition of the product-upgrade model, it is optimal for the monopolist to offer the upgraded product in the second period only if the waiting time $t$ for the upgraded product is less than $\frac{1}{3} \ln(\frac{\theta_H}{\theta_L^2})$.

Please note that the feasible condition of the product-upgrade model is $\max \{0, \ln(R)\} < t < \ln \left( R + \frac{\theta_H}{\theta_L} \right)$, which is derived from Proposition 2. Please refer to the proof in the Appendix.

The upgraded product has a higher quality and is targeted to H segment. However, H segment waits and buys the upgraded-quality product in the second period only if the waiting time is reasonable, i.e., less than $\frac{1}{3} \ln(\frac{\theta_H^2}{\theta_L^2})$ in case I. Otherwise, he will not wait and instead purchase the basic-quality product in the first period.

**Case II:** $R \in \left( \frac{\theta_L}{\theta_H + \theta_L}, e^t \right)$

When the degree of cannibalization is high, i.e., $R \in \left( \frac{\theta_L}{\theta_H + \theta_L}, e^t \right)$, recall that the monopolist targets H segment in the benchmark model. Note that the up-bound $e^t$ is a feasibility condition for the product-upgrade model, as stated in Proposition 2. Two strategies are available to the monopolist: (i) target H segment and offer a high quality product alone; (ii) target both segments and offer both the basic-quality product and its upgraded version one period after. We obtain the following result by comparing the profits of the two strategies in (15) and (4).

**Proposition 4:** In case II, it is optimal for the monopolist to offer the upgraded product if $R \in \left[ \frac{\theta_L}{\theta_H + \theta_L}, \max \left( \frac{\theta_L}{\theta_H + \theta_L}, R_2 \right) \right]$, where $R_2 = e^t - \sqrt{\frac{(e^{2t} - e^t) n_H \theta_H^2}{n_L \theta_L^2}}$. Otherwise, it is optimal to offer the high quality product alone.

The intuition is similar to that in Proposition 3. The degree of cannibalization needs to be low for the monopolist to be better off in offering the upgraded product. Specifically, $R < R_2$ and within the feasible conditions of case II.
In addition, the value of $R_2$ needs to be greater than $\frac{\theta_L}{\theta_H+\theta_L}$ for the monopolist to have an optimal region to offer the upgraded product. Otherwise, the optimal strategy for the monopolist is to target H segment in the first period and to offer no product upgrading afterwards. We obtain the following Corollary from this required constraint.

**Corollary 2:** *In case II, under the feasible condition of the product-upgrade model, it is optimal for the monopolist to offer the upgraded product in the second period only if the waiting time $t$ for the upgraded product is less than a threshold value $\bar{t}$.*

The expression of $\bar{t}$ can be found in the proof in the Appendix. The intuition of Corollary 2 is similar with that of Corollary 1. If the waiting time for the upgraded product is too long, H segment would buy the basic-quality product in the first period instead.

6. Summary

In this paper, we applied the self-selection approach to study optimal product upgrading strategies. We proposed a two-period product-upgrade model with the time-to-market of upgraded products incorporated. The model profit was benchmarked with that of a one-period model without product upgrading. We identified the conditions that are optimal to introduce upgraded products. The degree of cannibalization and the time-to-market were shown to be decisive factors. In particular, the product-upgrade model is optimal when the degree of cannibalization is small and when the waiting time is less than a threshold value. Otherwise, the one-period benchmark model is optimal.

Our work contributes to the literature of product development decisions. We provide detailed requirements of cannibalization and time-to-market for the introduction of upgraded products. The requirements are applicable for business practitioners, and can provide valuable insights and guidance in related business operations.

This paper can be extended in several directions. First, we assumed there was only a single attribute (quality) in the product. An important extension would be to include multiple attributes. The paper by Kim and Chhajed (2002) serves as a good reference. Second, like most other papers (Moorthy and Png, 1992; Desai et al., 2001; Mallik and Chhajed, 2006), we only considered a monopoly situation. The effect of competition would be interesting to explore.
Third, we exogenously assumed the time-to-market of upgraded products in our model. More valuable insights might be obtained by examining the timing factor as a decision variable. Finally, for simplicity, we did not consider production cost savings due to learning curve and design effort in the second period. Incorporating cost savings might be another interesting extension.

REFERENCES

APPENDIX

Proof of Proposition 1:
The monopolist’s optimal profit is \( \frac{(n_L + n_H)\theta_L^2}{4c} \) when both H and L segments purchase the product, and \( \frac{n_H\theta_H^2}{4c} \) when only H segment buys.

\[
\frac{(n_L+n_H)\theta_L^2}{4c} \geq \frac{n_H\theta_H^2}{4c} \iff 1 \geq \left[ \frac{n_H(\theta_H-\theta_L)}{n_L\theta_L} \right] \cdot \frac{\theta_H+\theta_L}{\theta_L} \iff 1 \geq R \cdot \frac{\theta_H+\theta_L}{\theta_L} \iff R \leq \frac{\theta_L}{\theta_H+\theta_L}.
\]

**Proof of Proposition 2:**

To ensure \( q_L^* > 0 \), condition \( 1 - e^{-t}R > 0 \) is required, from which we obtain \( R < e^t \).

For the product-upgrade model to be feasible, we also need the optimal qualities’ relationship in (13) to be \( q_H^* > q_L^* \), from which we obtain \( R > e^t - \frac{\theta_H}{\theta_L} \). This expression could be negative, therefore, we require \( R > \max [0, e^t - \frac{\theta_H}{\theta_L}] \).

Summarizing all the above relationships, we prove Proposition 2.

**Proof of Proposition 3:**

Under condition \( R \leq \frac{\theta_L}{\theta_H+\theta_L} \), we compare the two profits in (15) and (2). They are the profits when the monopolist offers the upgraded product and when there is no upgrading, respectively. From the profit relationship \( \frac{e^{-3t}n_H\theta_H^2}{4c} + \frac{n_L\theta_L^2}{4c} \cdot (1 - e^{-t}R)^2 \geq \frac{(n_L+n_H)\theta_L^2}{4c} \), we solve for the value of R. Two possible conditions are obtained: \( R \leq e^t - \sqrt{\frac{e^{2t}(n_L+n_H)\theta_L^2 - e^{-t}n_H\theta_H^2}{n_L\theta_L^2}} \) or \( R \geq e^t + \sqrt{\frac{e^{2t}(n_L+n_H)\theta_L^2 - e^{-t}n_H\theta_H^2}{n_L\theta_L^2}} \). The later is not feasible since it does not satisfy condition \( R < e^t \), which is required for the feasibility of the product-upgrade model.

Both conditions, \( R \leq \frac{\theta_L}{\theta_H+\theta_L} \) and \( R \leq e^t - \sqrt{\frac{e^{2t}(n_L+n_H)\theta_L^2 - e^{-t}n_H\theta_H^2}{n_L\theta_L^2}} \), need to be satisfied for the monopolist to offer the upgraded product. Thus, the overall condition is \( R \leq \min \left( \frac{\theta_L}{\theta_H+\theta_L}, e^t - \sqrt{\frac{e^{2t}(n_L+n_H)\theta_L^2 - e^{-t}n_H\theta_H^2}{n_L\theta_L^2}} \right) \).

**Proof of Corollary 1:**
From the restriction \( R_1 = e^t - \sqrt{\frac{e^{2t(n_L+n_H)\theta_H^2 - e^{-t}n_H\theta_H^2}}{n_L\theta_L^2}} \) to be positive, we obtain \( t < \frac{1}{3} \ln(\frac{\theta_H^2}{\theta_L^2}) \).

Please note the following feasibility conditions of the product-upgrade model should also be satisfied.

From constraint \( q_H > q_L^* \), we obtain \( t < \ln \left( R + \frac{\theta_H}{\theta_L} \right) \).

From constraint \( q_L^* > 0 \), we obtain \( t > \ln(R) \). The value \( \ln(R) \) could be negative, so we require \( t > \max \{ 0, \ln(R) \} \).

**Proof of Proposition 4:**

Similar with the proof of Proposition 3, under condition \( R \in \left( \frac{\theta_L}{\theta_H+\theta_L}, e^t \right) \), we compare the two profits in (15) and (4). From the profit relationship \( \frac{e^{-3t}n_H\theta_H^2}{4c} + \frac{n_L\theta_L^2}{4c} \left( 1 - e^{-t}R \right)^2 \geq \frac{n_H\theta_H^2}{4c} \), we solve for the value of \( R \), and obtain two possible conditions: \( R \leq e^t - \sqrt{\frac{(e^{2t-e^{-t}})n_H\theta_H^2}{n_L\theta_L^2}} \) or

\( R \geq e^t + \sqrt{\frac{(e^{2t-e^{-t}})n_H\theta_H^2}{n_L\theta_L^2}} \). Again, the later does not satisfy condition \( R < e^t \), thus is not feasible.

Both conditions, \( R \in \left( \frac{\theta_L}{\theta_H+\theta_L}, e^t \right) \) and \( R \leq e^t - \sqrt{\frac{(e^{2t(n_L+n_H)\theta_H^2-e^{-t}n_H\theta_H^2}}{n_L\theta_L^2}} \), need to be satisfied for the monopolist to offer the upgraded product. Please note that the value of \( e^t - \sqrt{\frac{(e^{2t(n_L+n_H)\theta_H^2-e^{-t}n_H\theta_H^2}}{n_L\theta_L^2}} \) is smaller than that of \( e^t \). In summary, the overall condition for Proposition 4 is \( R \in \left[ \frac{\theta_L}{\theta_H+\theta_L}, \max \left( \frac{\theta_L}{\theta_H+\theta_L}, e^t - \sqrt{\frac{(e^{2t-e^{-t}})n_H\theta_H^2}{n_L\theta_L^2}} \right) \right] \).

**Proof of Corollary 2:**

From the restriction \( e^t - \sqrt{\frac{(e^{2t-e^{-t}})n_H\theta_H^2}{n_L\theta_L^2}} > \frac{\theta_L}{\theta_H+\theta_L} \), we solve for \( t \), and obtain \( t < \bar{t} \), where \( \bar{t} \) is the \( t \) value that solves equation \( e^t - \sqrt{\frac{(e^{2t-e^{-t}})n_H\theta_H^2}{n_L\theta_L^2}} = \frac{\theta_L}{\theta_H+\theta_L} \). Please note that the feasibility conditions of the product-upgrade model should be satisfied too.