Teaching The Relationship Between Production And Cost

by

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Introduction

An essential microeconomic concept that many instructors find difficult
to teach is the connection between the separate parts of production theory
and the related cost concepts. Part of the teaching problem stems from the
fact that students have a variety of learning styles and for some an abstract
concept is only understandable if it can be incorporated into a structure that
the student can make "their own." Principles of Economics, as well as Inter-
mediate Microeconomics, textbooks typically compartmentalize production
theory and costs of production into separate chapters, or attempt to integrate
the material is such a way that the connection between production and cost
is either not apparent, or the transition is too muddled to be understandable.

What is the "best" way to teach the connection between production and
costs? In all likelihood there is no one "best" way. Some students learn eco-
nomic concepts through the use of "quantitative relationships," while others
are better able to understand when taught with a "visual" or "graphic" pre-
sentation. To incorporate both learning styles into the presentation of the production/cost relationship, we suggest a two-way approach: (1) the use of a computer spreadsheet that allows the student to alter the quantity of the variable input (labor) and compute the resulting production relationships and cost counterparts, and (2) a geometric approach, which derives from the production function the total, average, and marginal product curves and demonstrates a geometric method of deriving the cost counterparts of total, average, and marginal cost. Both methods start with the same basic short-run production and cost functions presented below.

The Short-Run Production Function

The structure that represents a business firm begins with data generated from a short-run production function. The production function shows the technical relationship between resource inputs and physical output. In the short-run at least one significant input (capital), is constant (fixed), while at least one input (labor), can be changed (variable). The production function can be written as:
\[ O = 1.3L^2 - 0.05L^3 \] (1)

In this example capital investment \( \overline{C} \), an input that is not easily changed, is the fixed input.\(^1\) Units of labor \( L \), an input more easily altered, is the variable input and can be increased from 0 up to any positive amount. The production, that results from combining the variable input labor with the fixed input capital is physical output \( O \). The production function used has the conventional properties of increasing returns,\(^2\) followed at some level of output, by diminishing returns\(^3\) to the variable factor.

The average product of labor \( AP_L \) is calculated by dividing the production function by the quantity of labor. The marginal product of labor \( MP_L \) is calculated by taking the first derivative of the production function. Both

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\(^1\)Capital is not shown in the production function because we are demonstrating how output \( O \) will change as only labor \( L \) is altered.

\(^2\)Increasing returns may come about when less labor is used to operate the firm than what the firm was designed to efficiently use. The multiplicity of jobs performed by each individual and the time lost in changing from one task to another create a situation where equal increases in labor used bring about successively greater increases in total output (production), up to a point.

\(^3\)The fundamental concept of production, the law of diminishing returns, can be stated thus: "adding equal units of a variable input (labor) to a fixed input (capital) will cause total output to increase; but beyond some point, the resulting output increases will become smaller and smaller."
resulting equations are shown below:

\[ AP_L = 1.3L - 0.05L^2 \]  \hspace{1cm} (2)

\[ MP_L = 2.6L - 0.15L^2 \]  \hspace{1cm} (3)

The Short-Run Cost Functions

Cost functions are calculated according to standard formulas: Total costs \( TC \) are the sum of total fixed cost \( TFC \) and total variable costs \( TVC \). Total variable cost \( TVC \) is the reciprocal of the average product of labor \( AP_L \) multiplied by the unit labor cost \( P_L \), while marginal cost \( MC \) is the reciprocal of the marginal product of labor \( MP_L \) multiplied by the unit labor cost \( P_L \). These equations are shown below:

\[ TC = TFC + TVC \]  \hspace{1cm} (4)

\[ TVC = P_L \left( \frac{1}{AP_L} \right) \]  \hspace{1cm} (5)

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4 This relationship can be demonstrated by the following: \( AVC = \frac{TVC}{Q} = \frac{PL \cdot Q}{Q} = PL \left( \frac{Q}{Q} \right) = PL \left( \frac{1}{AP_L} \right), since \frac{Q}{Q} = AP_L \)

5 This relationship can be demonstrated by the following: \( MC = \frac{\delta TC}{\delta Q} = \frac{\delta TVC}{\delta Q} = PL \left( \frac{1}{MP_L} \right), since \frac{\delta Q}{\delta L} = MP_L \)
\[ MC = P_L \left( \frac{1}{MP_L} \right) \]  

(6)

Spreadsheet Presentation of Production and Cost

The spreadsheet presentation shown on Table 1 begins with the placement of column titles for capital, labor, and output, over the first three spreadsheet columns. The average and marginal products are added, followed by costs. Cost schedules are total fixed, total variable, total costs, average variable, and marginal costs. The reciprocal of the average product and the marginal product are also calculated for use in the plotting of the schedule in the following graphic presentation (Panel 3 of Figure 2). This schedule, which is a rectangular hyperbola, is an effective device for the translation of product curves to cost curves\(^6\).

Labor input begins at zero and increases by increments of either 1 or 2 until either 24 or 12 cells under the column labeled “Labor” are filled. The ending labor input should be 22 units. Capital, the fixed input, is 10 units in each of the cells. Using the equation for output, or the production function,

\(^6\)This device was developed by Gene C. Uselton at Texas A&M University. "Graphically Deriving Costs From Production Functions," Great Ideas for Teaching Economics, ed. Ralph T. Byrns and Gerald W. Stone, 2nd Ed., Dallas: Scott, Foresman and Company, 1984, pp. 120-122
\( O = 1.3L^2 - .05L^3 \), the output is calculated, and that calculation is copied, with the "copy" command to each of the other cells in the Output column\(^7\). The values, calculated by the spreadsheet for output, reflect the expected shape of a production function with increasing returns to the variable input, followed by diminishing returns. The output data are the data points that will be used in the following construction of the graphic presentation of production and costs.

The average product of labor is calculated by dividing the output function by the quantity of labor input, and the marginal product of labor is the first derivative of the output function. Equations (2) and (3) show how the average product of labor and the marginal product of labor are calculated. A list of cell formulas for calculating product and cost is found in Table 2.

Capital, the fixed input costs $50 per unit, and because 10 units of capital are used, the Total Fixed Cost is $500 throughout the range of production. Total Variable Costs are determined by multiplying the number of units of labor by the wage rate of labor which is $90 per labor unit. Total Cost is the sum of Total Fixed Cost and Total Variable Cost at the various levels of

\(^7\)All spreadsheets have similar features and commands. See the command documentation for your specific spreadsheet.
output.

Average Total Cost is Total Cost divided by output, and Average Variable Cost is Total Variable Cost divided by output. The calculations must begin at a level of output greater than zero (the cell in the second row) to avoid division by zero. The calculations for Average Variable and Average Total Costs are copied to other cells using the spreadsheet “copy” command.

Marginal Cost, the derivative of Total Cost, also begins in the second cell, and should be thought of as occupying a location half-way between two adjacent cells because Marginal Cost requires the incremental change in Total Cost. The values for other cells are placed using the “copy” command.

An advantage to the use of this product and cost spreadsheet is its ability to instantly show the result of changes in any of the variables that make up the basic functions. The price of inputs, the quantity of inputs, or the parameters of the functions may be changed. With each such change the spreadsheet is recalculated, and the result is available in an instant.

A logical extension would be the inclusion of demand and revenue functions, used with these cost relationships to show the calculation of profit. The rapid recalculation of profit after change in any revenue variable demonstrates to the student the sensitivity of those variables on the firms profitability.
### Table 1

**PRODUCTION AND COSTS FOR A BUSINESS FIRM**

<table>
<thead>
<tr>
<th>Capital</th>
<th>Labor</th>
<th>Output</th>
<th>$AP_L$</th>
<th>$MP_L$</th>
<th>TFC</th>
<th>TVC</th>
<th>TC</th>
<th>ATC</th>
<th>AVC</th>
<th>MC</th>
<th>$1/AP_L$</th>
<th>$1/MP_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>N.A.</td>
<td>N.A.</td>
<td>$500$</td>
<td>0</td>
<td>$500$</td>
<td>N.A.</td>
<td>N.A.</td>
<td>N.A.</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>5</td>
<td>2.4</td>
<td>4.6</td>
<td>500</td>
<td>$180$</td>
<td>680</td>
<td>$141.67$</td>
<td>$37.50$</td>
<td>$19.57$</td>
<td>0.42</td>
<td>0.22</td>
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<tr>
<td>10</td>
<td>4</td>
<td>18</td>
<td>4.4</td>
<td>8.0</td>
<td>500</td>
<td>360</td>
<td>860</td>
<td>48.86</td>
<td>20.45</td>
<td>11.25</td>
<td>0.23</td>
<td>0.13</td>
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<tr>
<td>10</td>
<td>6</td>
<td>36</td>
<td>6.0</td>
<td>10.2</td>
<td>500</td>
<td>540</td>
<td>1040</td>
<td>28.89</td>
<td>15.00</td>
<td>8.82</td>
<td>0.17</td>
<td>0.10</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>58</td>
<td>7.2</td>
<td>11.2</td>
<td>500</td>
<td>720</td>
<td>1220</td>
<td>21.18</td>
<td>12.50</td>
<td>8.04</td>
<td>0.14</td>
<td>0.09</td>
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<tr>
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<td>10</td>
<td>80</td>
<td>8.0</td>
<td>11.0</td>
<td>500</td>
<td>900</td>
<td>1400</td>
<td>17.50</td>
<td>11.25</td>
<td>8.18</td>
<td>0.13</td>
<td>0.09</td>
</tr>
<tr>
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<td>12</td>
<td>101</td>
<td>8.4</td>
<td>9.6</td>
<td>500</td>
<td>1080</td>
<td>1580</td>
<td>15.67</td>
<td>10.71</td>
<td>9.37</td>
<td>0.12</td>
<td>0.10</td>
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<td>118</td>
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<td>7.0</td>
<td>500</td>
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<td>1760</td>
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<td>10.71</td>
<td>12.86</td>
<td>0.12</td>
<td>0.14</td>
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<td>128</td>
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<td>3.2</td>
<td>500</td>
<td>1440</td>
<td>1940</td>
<td>15.16</td>
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<td>0.13</td>
<td>0.31</td>
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<td>500</td>
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<td>16.36</td>
<td>12.50</td>
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<td>6.0</td>
<td>-8.0</td>
<td>500</td>
<td>1800</td>
<td>2300</td>
<td>19.17</td>
<td>15.00</td>
<td></td>
<td></td>
<td>0.17</td>
</tr>
<tr>
<td>10</td>
<td>22</td>
<td>97</td>
<td>4.4</td>
<td>-15.4</td>
<td>500</td>
<td>1980</td>
<td>2480</td>
<td>25.82</td>
<td>20.45</td>
<td></td>
<td></td>
<td>0.23</td>
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Table 2

SPREADSHEET CELL FORMULAS

<table>
<thead>
<tr>
<th>Column Heading</th>
<th>Cell</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>A&lt;sub&gt;i&lt;/sub&gt;</td>
<td>10</td>
</tr>
<tr>
<td>Labor</td>
<td>B&lt;sub&gt;i&lt;/sub&gt;</td>
<td>L&lt;sub&gt;x&lt;/sub&gt;</td>
</tr>
<tr>
<td>Output</td>
<td>C&lt;sub&gt;i&lt;/sub&gt;</td>
<td>(1.3*&lt;sup&gt;B&lt;sub&gt;i&lt;/sub&gt;&lt;/sup&gt;-2)-(0.05*&lt;sup&gt;B&lt;sub&gt;i&lt;/sub&gt;&lt;/sup&gt;-3)</td>
</tr>
<tr>
<td>AP&lt;sub&gt;L&lt;/sub&gt;</td>
<td>D&lt;sub&gt;i&lt;/sub&gt;</td>
<td>(1.3*&lt;sup&gt;B&lt;sub&gt;i&lt;/sub&gt;&lt;/sup&gt;)-(0.05*&lt;sup&gt;B&lt;sub&gt;i&lt;/sub&gt;&lt;/sup&gt;-2)</td>
</tr>
<tr>
<td>MP&lt;sub&gt;L&lt;/sub&gt;</td>
<td>E&lt;sub&gt;i&lt;/sub&gt;</td>
<td>(2.6*&lt;sup&gt;B&lt;sub&gt;i&lt;/sub&gt;&lt;/sup&gt;)-(0.1.5*&lt;sup&gt;B&lt;sub&gt;i&lt;/sup&gt;&lt;/sup&gt;-2)</td>
</tr>
<tr>
<td>TFC</td>
<td>F&lt;sub&gt;i&lt;/sub&gt;</td>
<td>+A&lt;sub&gt;i&lt;/sub&gt;*50</td>
</tr>
<tr>
<td>TVC</td>
<td>G&lt;sub&gt;i&lt;/sub&gt;</td>
<td>+B&lt;sub&gt;i&lt;/sub&gt;*90</td>
</tr>
<tr>
<td>TC</td>
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<td>+F&lt;sub&gt;i&lt;/sub&gt; + G&lt;sub&gt;i&lt;/sub&gt;</td>
</tr>
<tr>
<td>ATC</td>
<td>I&lt;sub&gt;i&lt;/sub&gt;</td>
<td>H&lt;sub&gt;i&lt;/sub&gt;/C&lt;sub&gt;i&lt;/sub&gt;</td>
</tr>
<tr>
<td>AVC</td>
<td>J&lt;sub&gt;i&lt;/sub&gt;</td>
<td>(1/D&lt;sub&gt;i&lt;/sub&gt;*90)</td>
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<tr>
<td>MC</td>
<td>K&lt;sub&gt;i&lt;/sub&gt;</td>
<td>(1/E&lt;sub&gt;i&lt;/sub&gt;*90)</td>
</tr>
<tr>
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<td>L&lt;sub&gt;i&lt;/sub&gt;</td>
<td>1/D&lt;sub&gt;i&lt;/sub&gt;</td>
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<tr>
<td>1/MP&lt;sub&gt;L&lt;/sub&gt;</td>
<td>M&lt;sub&gt;i&lt;/sub&gt;</td>
<td>1/E&lt;sub&gt;i&lt;/sub&gt;</td>
</tr>
</tbody>
</table>

Where:

\[
L = \text{units of Labor}
\]
\[
x = i - 1
\]
\[
i = \text{Rows increasing from 1 to 23}
\]
\[
+ = \text{Addition}
\]
\[
- = \text{Subtraction}
\]
\[
* = \text{Multiplication}
\]
\[
/ = \text{Division}
\]
\[
^ = \text{Exponent}
\]
Graphic Presentation of Production and Cost

Production and Total Variable Cost:

It is a relatively simple matter to geometrically construct a Total Variable Cost curve from the production function. Panel 1 of Figure 1 graphically depicts the production function \( O = 1.3L^2 - .05L^3 \) used in the spreadsheet example. Units of labor \( (L) \) are measured horizontally, while output \( (O) \) is measured along the vertical axis. By increasing the units of labor from zero up to 22 units the production function shown in Panel 1 is traced out. This production function provides the basis for the construction of all cost functions because it not only tells how much output will be produced with various amounts of labor, it also depicts how much labor it takes to produce various levels of output. Knowing how many units of labor it takes to produce each output level provides the basis of how much it will cost to produce the output.

To use the production function to derive the Total Variable Cost \((TVC)\) function requires only that we convert the units of labor require to produce each output level to the cost of labor to produce each output level. Since the cost of production is dependent on the desired level of output, \((TVC)\)
Figure 1
The Relationship Between Production
And Total Variable Cost

Panel 1

Panel 4

Q = 1.3L - 0.5L^2

Panel 2

Panel 3

TVC = P_L * L
needs to be shown on the vertical axis, the standard axis for measuring the
dependent variable. Output \((O)\), the dependent variable in the production
function and shown vertically, now becomes the independent variable is needs
to be shown horizontally. Panels 2 and 4 of Figure 1 allow the transformation
of the production function in Panel 1 to the Total Variable Cost function in
Panel 3.

Panel 2 of Figure 1 shows the units of labor \((L)\) on the horizontal axis and
the associated cost of the various units of labor on the vertical axis. Using a
unit price of labor \((P_L)\) of $90, the cost of 2 units of labor will be $180, while
6 units will cost $540 and 20 units will cost $1,800. Thus by knowing how
many units of labor are required to produce some level of output, shown in
Panel 1, we can look to Panel 2 and find the Total Variable Cost of labor.

Panel 4 of Figure 1 is simply a mechanism for transforming the units
of output, measured vertically in Panel 1, a horizontal measurement. With
this conversion it is now possible to trace out a Total Variable Cost curve
in Panel 3, from the information contained in Panel 1. For example, the
production function indicates that when 6 units of labor are employed the
resulting output will be 36. The cost of 6 units of labor is $540 and plotting
in Panel 3 the coordinates 36, $540 gives a point on the Total Variable Cost
curve. Similarly, by taking any other point on the production function it is possible to find the cost of labor associated with that level of production and to continue to trace out in Panel 3 the variable cost associated with each level of production, which is the Total Variable Cost curve.

The Relationship Between Average Product/Average Cost and Marginal Product/Marginal Cost:

Using the same short-run production equation $O = 1.3L^2 - 0.05L^3$ used in the spreadsheet example, we plot in Panel 1 of Figure 2 the data generated in the spreadsheet by that equation. In a Panel 2 immediately below the total product ($O$), or production function, the Average Product of Labor ($AP_L$) and the Marginal Product of Labor ($MPL$) are plotted from the spreadsheet calculations.

The transmission linkage from the product curves, $O$, $AP_L$, $MPL$, to Average Variable Cost ($AVC$) and Marginal Cost ($MC$) proceeds from two directions. Levels of output are taken from the production function in Panel 1 and, through a 45 degree turning axis, directed to the horizontal axis in Panel 6. Average Product ($AP_L$) and Marginal Product ($MPL$) curves are converted to Average and Marginal Costs, numerically, by multiplying the reciprocal of product curves times the unit wage rate ($P_L$) of $100$. Graphi-
Figure 2
The Relationship Between Average Product/Average Cost
And Marginal Product/Marginal Cost
cally, the average and marginal product numbers multiplied by any constant, such as the unit cost of labor, plots as a rectangular hyperbola. This transformation is depicted in Panel 3 of Figure 2. Panel 4 of Figure 2 is simply a turning axis which will allow the units cost of production to be measured on the vertical axis.

Panel 5 allows Output from the production function, and Average and Marginal Cost to be brought together in one panel yielding a diagram that depicts the Average and Marginal Costs curves associated with each level of production. The curves in Panel 5 have the standard shapes that are expected when diminishing returns exist in the production function. Additionally, Figure 2 depicts in a graphic way exactly how Average Variable Cost relates to Average Product and how Marginal Cost relates to Marginal Product.

**Conclusion**

The purpose of this paper was two fold. First to allow the student who learns by “doing” a mechanism to workout using a spreadsheet approach

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8The transformation plots as a rectangular hyperbola because the numbers on the horizontal axis are simply the reciprocal of the numbers plotted numerically times some constant, the unit wage rate. Since in the example the unit wage is set at $100, multiplying any number on the horizontal axis by its counterpart of the vertical axis will equal $100. Altering the wage rate will move the location of the transformation, but the shape will remain a rectangular hyperbola.
the relationship between production and the associated cost functions. Second, for the visual learner we used a diagrammatic method of converting production to the associated costs of production. Each of these methods have strengths and weaknesses. Some students will find one method more appealing than the other. But the two methods integrated together can be a powerful tool for allow students to become familiar with the use of spreadsheets, and how formulas can be incorporated into the cells to make the necessary calculations. And for the visual learner we have introduced a mechanism that will allow the student to “see” the relationships that exist between production and the associated costs of production.