

### Sam Houston State University Department of Economics and International Business Working Paper Series

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SHSU Economics & Intl. Business Working Paper No. 10-08 December 2010

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#### Abstract

The purpose of this paper is to construct a dynamic stochastic production frontier incorporating the sluggish adjustment of inputs, to measure the speed of adjustment of output, and to compare the technical efficiency estimates from this dynamic model to those from a static model. By assuming instantaneous adjustment of all inputs, a static model may underestimate technical efficiency of a production unit in the short-run. However, in this paper I show that under the assumption of similar adjustment speed for all inputs, a linear partial adjustment scheme for output characterizes the dynamic production frontier. The dynamic frontier with timeinvariant technical efficiency is estimated using the system GMM (generalized method of moments) estimator. Applying the model and estimation method on a panel dataset spanning nine years of data on private manufacturing establishments in Egypt, I find that 1) the speed of adjustment of output is significantly lower than unity, 2) the static model underestimates technical efficiency by 4.5 percentage points on average, and 3) the ranking of production units based on their technical efficiency measures changes when the lagged adjustment process of inputs is taken into account.

Key words: Adjustment of inputs, dynamic panel data models, stochastic production frontier, time-invariant technical efficiency.

JEL Classification: C23, D24, L60

## 1. Introduction

Production frontier estimation and the measurement of technical efficiency of production systems have been important areas of research for more than half a century. Following the pioneering work of Aigner, Lovell, and Schmidt (1977) and Meeusen and Broeck (1977), who independently proposed the estimation of stochastic production frontier, this field has further grown with important contributions by many researchers (see Schmidt and Lovell (1979), Jondrow et al. (1982)). These studies have posited two main causes for the deviation of actual output from the maximum possible output (potential output), given the inputs. A part of this deviation is attributed to the symmetric random shocks to a production system that are not under the control of a producer (e.g., uncertainty about the weather, or input market conditions). The other reason for the failure to produce the potential output, given a set of inputs, is the presence of technical inefficiency caused by factors such as managerial error and coordination failures. Accordingly, a firm is said to be technically inefficient if it produces below the production frontier, and the corresponding technical inefficiency is measured by the deviation of the actual output from the frontier, after accounting for the random shocks to the system.

The literature has expanded to include both time-invariant and timevarying technical efficiency measures (see Cornwell, Schimdt and Sickles (1990); Kumbhakar (1990); Kumbhakar (1991); Battese and Coelli (1992); Lee and Schmidt (1993); Ahn, Lee, and Schimdt (1994); and Kumbhakar, Heshmati, and Hjalmarsson (1997)), as well as cross-sectional and panel data models of stochastic frontier estimation (see Schmidt and Sickles (1984)). A general discussion on the measurement of productive efficiency and the related literature can be found in Lovell (1996), Kumbhakar and Lovell (2000), and Coelli et al. (2005).

Most of the existing studies on stochastic frontiers and technical efficiency are based on the assumption that when an input is introduced into the production system, it immediately contributes to production at its maximum possible level. However, once introduced to a production system, an input may require some time for adjustment within the system. Possible causes include quasi-fixity of inputs, time needed to learn, and/or different contractual bindings. Given the time for adjustment, it may not be possible for a firm to catch up with the production frontier instantaneously following the introduction of a new input, even in the absence of any other source of inefficiency. A vast literature on the source, structure, size and specification of adjustment costs (Lucas (1967a, 1967b); Treadway (1971); and Hamermesh and Pfann (1996)) has established the importance of the adjustment process in the theory of production.

Consequently, behind the productivity change of a firm, a dynamic process is likely to be at work in terms of input adjustment. This dynamic adjustment process is a natural phenomenon of any production system and thus the shortfall in the output that results does not really represent inefficiency of the production unit. The adjustment process of inputs is rather an inherent characteristic of any production system that cannot completely be controlled by the producers. Hence, a static production frontier model that ignores the effect of input adjustment on output may misspecify the process of output generation. Consequently, technical efficiency measures from such a misspecified model are likely to be biased.

Among studies that have considered sluggish adjustment of inputs, Ahn, Good, and Sickles (1998, 2000) and Hultberg, Nadiri, and Sickles (1999) assume that technical innovations introduced at the beginning of a period are only partially adopted. Thus, according to their assumption, the actual productivity in any period depends on the actual productivity in the previous period as well as on the productivity level which could be achieved if the technology innovations were instantaneously adopted. The speed of adjustment plays a crucial role in determining how quickly productivity gains are realized and in turn, implies that the technical efficiencies of production units are autoregressive. Thus, the current output depends on the current inputs and on both last period's output and inputs. Other studies that incorporate dynamic adjustment include Ayed-Mouelhi and Goaied (2003), Kumbhakar, Hesmati, and Hjalmarsson (2002), and Asche, Kumbhakar, and Tveteras (2008).

Sluggish adjustment of inputs not only affects the adoption of technological innovations, but can also affect the whole production process by preventing output from reaching its maximum possible level. Further, ranking of the production units is also likely to be altered in the presence of lagged adjustment of inputs.

The present paper is also motivated by the idea that inputs and changes in production plans are sluggishly adopted but takes the next logical step that due to the sluggish adjustment of inputs, output follows a partial adjustment scheme. Current output depends on the last period's output, and the potential output (output that could be achieved with instantaneous adjustment of inputs in a fully efficient production system). Thus, the theoretical model developed in this paper presents a production process from a perspective similar to but different from that discussed in Ahn, Good, and Sickles (1998, 2000), and Hultberg, Nadiri, and Sickles (1999). The model also portrays an idea similar to Hultberg, Nadiri, and Sickles (2004), which highlights the importance of the productivity gap in determining the growth rate of output. The principal objective of this paper is to measure technical efficiency using a dynamic, stochastic production frontier incorporating lagged adjustment of inputs, and to compare the resulting estimates of time-invariant technical efficiency of production units with the estimates from a static production model assuming instantaneous adjustment of all inputs. For this purpose, I use a panel dataset on private manufacturing establishments in Egypt from the Industrial Production Statistics of the Central Agency for Public Mobilization and Statistics (CAPMAS).

The remainder of this paper is organized as follows. The main theoretical and econometric models are presented in sections 2 and 3, respectively. Section 4 elaborates on the estimation methods. Results from the empirical analysis are described in section 5, and finally, section 6 presents concluding remarks.

### 2. Theoretical Model

The dynamic production model is based on the following three assumptions. First, the speeds of adjustment of inputs are similar for all inputs at every time period. In reality, different inputs may have different speeds of adjustment, which may vary with time as well. However, I focus on the simplified production model as the base model in this paper.<sup>1</sup> Second, the output is generated by a partial adjustment scheme, i.e., the change in actual output between two periods is a fraction of the desired change in output in that period. Third, the speed of adjustment of output is determined by the speed of adjustment of inputs. Therefore, the speed of adjustment of inputs and output are similar in nature.

After introduction of inputs, it is logical to have a time lag before they produce at their maximum possible level. Therefore, it is likely that a newer input will

<sup>&</sup>lt;sup>1</sup> Extending the basic model to a more general one with input specific and time varying speeds of adjustment of inputs is an interesting and open area of future research.

contribute less to the output than the older ones. For example, a worker who was hired a month ago would be more familiar with the production process than a worker who was hired a day ago. Hence, the new worker's contribution would be less at the beginning.

The change in actual output between any two periods is a combined result of contribution of new inputs, a part of which is adjusted during the period, and contribution of a part of the old inputs that adjusts in that period. Therefore, during the adjustment process of inputs, the current output is higher than previous period's output, but lower than the potential output, when the potential output is increasing over time. Let us refer to the change in output that is needed in any period to catch up with the potential output, as the 'desired change' in output. The difference between the actual and the desired change in output depends on the speed of adjustment of inputs.

To further analyze the production model, let us consider a general production function for the potential output  $y_{it}^*$  of firm *i* that uses a vector of inputs  $x_{it}$  at time *t*.

$$y_{it}^* = f(x_{it}, \beta)$$
 (2.1)

where i = 1,...,N denotes the production unit, t = 1,...,T represents the time periods, and  $\beta$  is the technology parameter. Let  $y_{it}$  be the actual output produced by firm *i* at time *t*, and let  $\lambda$  ( $0 \le \lambda \le 1$ ) be the speed of adjustment of inputs.

In the initial period of production, the actual output is only  $\lambda$  fraction of the potential output. From next period onwards, not only  $\lambda$  fraction of the potential output in that period is produced, but also  $\lambda$  fraction of the gap between the potential output and the previous period's output is covered. If the speed of adjustment is lower than unity, then the actual output will be lower than the potential output. Moreover, the higher is the speed of adjustment of inputs, the lower is the deviation of actual output from the potential output, and the potential output is exactly the actual one when the speed of adjustment is unity, i.e., when inputs are instantaneously adjusted in the production system. Thus, in the initial perod,

$$y_{it} = \lambda y_{it}^* \tag{2.2}$$

and 
$$y_{it}^* - y_{it} = (1 - \lambda) y_{it}^*$$
 (2.3)

Therefore, the dynamic process of output generation can be represented by -

$$y_{it+1} = \lambda y_{it+1}^* + \lambda (1-\lambda) y_{it}^*$$
 (2.4)

or, 
$$y_{it+1} = \lambda y_{it+1}^* + (1-\lambda) y_{it}$$
 (2.5)

or, 
$$y_{it} = \lambda y_{it}^* + (1 - \lambda) y_{it-1}$$
 (2.6)

Using (2.6) for output produced in each period, the partial adjustment scheme of output as given in (2.6) can further be restated as follows-

$$y_{it} = \lambda f(x_{it}, \beta) + (1 - \lambda)(\lambda f(x_{it-1}, \beta) + (1 - \lambda)y_{i(t-2)})$$
  
or,  $y_{it} = \lambda f(x_{it}, \beta) + \lambda (1 - \lambda)f(x_{it-1}, \beta) + \lambda (1 - \lambda)^2 f(x_{it-2}, \beta) + ...$  (2.7)

The partial adjustment scheme for actual output at time *t* demonstrates that the current output depends on the current and past inputs. With a speed of adjustment that is less than unity, the most recent past of input usage receives the greatest weight in determining the current output, and influence of past inputs will fade out uniformly with the passage of time. Therefore, the distant past receives arbitrarily small weight. This refers to the fact that the unadjusted part of an input continues to adjust and contribute to production. With the passage of time, an input is almost fully adjusted, and hence only a very small unadjusted part remains to increase its contribution to the production process.

## 3. Econometric Model

The potential output is a hypothetical characterization of the maximum possible output and is not observed in reality. The actual output is generally above or below the potential output because a production system is exposed to random shocks that may positively or negatively affect production plans. Moreover, a production unit is likely to suffer from technical inefficiency that may lower the actual output. The stochastic version of (2.6), which is more realistic, considers a composite error term that accounts for the random shocks to a production unit, and the technical inefficiency of that unit. I obtain the stochastic versions of the dynamic output generation process (2.6) by considering a composite error term ( $\varepsilon_{ii}$ ) consisting of symmetric random shocks  $v_{ii}$  to firm *i* at time *t*, and the producer specific effects,  $u_i$ , that determine the technical inefficiency of each production unit and are constant over time<sup>2</sup>.

Therefore, following (2.6), the production model is -

$$y_{it} = (1 - \lambda) y_{i(t-1)} + \lambda y_{it}^* + \varepsilon_{it}$$
(3.1)

where i = 1,...,N, and t = 1,...,T,  $\varepsilon_{it} = v_{it} - u_i$ , and  $u_i \ge 0$  captures the producer specific, time-invariant, non-negative inefficiency effects for production unit *i*. In a more general set up where the technical efficiency varies with time  $(u_{it})$ , the difference between the potential output and the short run actual output captures the inefficiency of the production model. Assuming that the effects of random shocks are similar on  $y_{it}^*$  and  $y_{it}$ ,

 $<sup>^{2}</sup>$  The model can further be generalized to estimate time-varying technical efficiency of each production unit. However, discussion on the more general case is beyond the scope of this paper.

$$u_{it} = y_{it}^* - y_{it}$$
  
=  $y_{it}^* - (1 - \lambda) y_{it-1} - \lambda y_{it}^*$   
=  $(1 - \lambda) y_{it}^* - (1 - \lambda) (y_{it-1}^* - u_{it-1})$   
=  $(1 - \lambda) u_{it-1} + (1 - \lambda) (y_{it}^* - y_{it-1}^*)$ 

Thus, when the output is generated by a partial adjustment scheme, the technical inefficiency in any period depends on the last period's inefficiency, the speed of adjustment of inputs, and the change in potential output. The change in potential output may also be referred to as the "potential output gap". If the technical inefficiency is constant over time, as assumed in this paper, then it depends on the speed of adjustment and the "potential output gap". This result is not surprising in view of the fact that Hultberg, Nadiri, and Sickles (2004) discuss the process of capital evolution where they show that the growth rate of output in any period depends on the speed of technology adoption and the productivity gap in the last period.

Let us consider a Cobb-Douglas function for the production of potential output<sup>3</sup>-

$$\ln y_{it}^* = \beta_0 + \sum_{m=1}^M \beta_m \ln x_{mit} + \sum_{t=2}^T \delta_t D_t , \qquad (3.2)$$

where i = 1,...,N denotes the production unit, t = 1,...,T represents the time periods, m = 1,...,M represents the inputs used in production,  $\beta_m$  is the elasticity of the *m*th input, and  $\beta_0$  is the intercept of the potential production frontier. I introduce the time dummy variables  $D_t$  in the production model to incorporate the pure technological change as proposed by Baltagi and Griffin (1988). Thus, no specific structure is imposed on the behavior of the technological change.  $\delta_t$ captures the effect of technological changes on the potential output. Then, the

<sup>&</sup>lt;sup>3</sup> The analysis is valid for more general production functions.

dynamic stochastic production frontier that incorporates the sluggish adjustments of inputs<sup>4</sup> and time-invariant technical inefficiency, is given by-

$$\ln y_{it} = (1 - \lambda) \ln y_{i(t-1)} + \lambda (\beta_0 + \sum_{m=1}^M \beta_m \ln x_{mit} + \sum_{t=2}^T \delta_t D_t) + \varepsilon_{it}$$
(3.3)

where i = 1,...,N, t = 1,...,T, m = 1,...,M. In (3.3),  $\varepsilon_{it} = v_{it} - u_i$ , and  $u_i \ge 0$ captures the producer specific, time-invariant, non-negative inefficiency effects for production unit *i* with  $E(u_i) = \mu$ , and variance  $\sigma_u^2$ .  $v_{it}$  are the random shocks to the production unit *i* at time *t*, with zero mean and variance  $\sigma_v^2$ . The time dummies,  $D_t$ , have value equals unity for year *t* and zero otherwise. I further assume that  $\lambda \beta_0 - \mu = \beta_0^*$ ,  $u_i^* = u_i - \mu$  such that  $u_i^* \sim iid(0, \sigma_u^2)$  and the stochastic production model can be written as

$$\ln y_{it} = (1 - \lambda) \ln y_{i(t-1)} + \beta_0^* + \lambda (\sum_{m=1}^M \beta_m \ln x_{mit} + \sum_{t=2}^T \delta_t D_t) + v_{it} - u_i^*$$
(3.4)

The standard structure of the error component as discussed in Blundell and Bond (1998) is also maintained as follows-

- 1.  $u_i^*$  is uncorrelated with  $v_{ii}$ , i.e.  $E(v_{ii}u_i^*) = 0$  for all i = 1,...,N, and t = 1,...,T.
- 2.  $v_{it}$  is serially uncorrelated, i.e.  $E(v_{it}v_{is}) = 0$  for all i = 1, ..., N, and  $t \neq s$ .
- 3.  $E(y_{i1}v_{it}) = 0$  for i = 1, ..., N, and t = 2, ..., T.

<sup>&</sup>lt;sup>4</sup> Equation (3.3) can also be expressed in the form of (2.6) which demonstrates the fact that a fraction,  $\lambda$ , of an input  $x_{mi(t-k)}$  that is introduced by firm *i* in the period t - k (0 < k < t), contributes to the output in that period. In period t - k + 1,  $\lambda$  fraction of the remaining  $(1 - \lambda) x_{mi(t-k)}$  contributes to the output, and again  $\lambda$  fraction of the unadjusted  $(1 - \lambda)^2 x_{mi(t-k)}$  contributes to output in t - k + 2. Following this process,  $\lambda$  fraction of  $(1 - \lambda)^k x_{mi(t-k)}$  contributes to output at time *t*. Therefore, the marginal effects of current inputs are higher than those for the inputs from previous periods.

In the dynamic model (3.3), the parameter  $\lambda$ , which is invariant over time, producer, and inputs, reflects the fraction of the desired change in output that is realized in any period. Following Schmidt and Sickles (1984), the most efficient production unit in the sample is assumed to be 100% efficient, and technical efficiency of other units are measured relative to the best-practice frontier -

$$TE_{i} = \exp\{-(\max_{i}(-\hat{u}_{i}^{*}) - (-\hat{u}_{i}^{*}))\}$$
(3.5)

where a consistent estimator of  $\hat{u}_i^*$  is given by -

$$\hat{u}_{i}^{*} = \frac{-1}{T-1} \sum_{t=2}^{T} \left( \ln y_{it} - (1-\hat{\lambda}) \ln y_{i(t-1)} - \hat{\beta}_{0}^{*} - \hat{\lambda} \sum_{m=1}^{M} \hat{\beta}_{m} \ln x_{mit} - \hat{\lambda} \sum_{t=2}^{T} \hat{\delta}_{t} D_{t} \right)$$
(3.6)

The conventional static specification of the stochastic production frontier assumes instantaneous adjustment of inputs while catching up with the potential output and hence  $\lambda = 1$  for the static version of (3.3). Formally, the static production frontier is given by -

$$\ln y_{it} = \beta_0 + \sum_{m=1}^{M} \beta_m \ln x_{mit} + \sum_{t=2}^{T} \delta_t D_t - \eta_i + \upsilon_{it}$$
(3.7)

Here,  $\eta_i$  represents the non-negative producer specific inefficiency effects. Therefore, the technical efficiency is measured from (3.7) as

$$T\tilde{E}_{i} = \exp\{-(\max_{i}(-\hat{\eta}_{i}^{*}) - (-\hat{\eta}_{i}^{*}))\}$$
(3.8)

where  $\eta_i^* = \eta_i - E(\eta_i)$ ,  $\beta_0^* = \beta_0 - E(\eta_i)$ ,  $\eta_i^* \sim iid(0, \sigma_\eta^2)$ , and  $\upsilon_{ii} \sim iid(0, \sigma_v^2)$ . If the producer specific effects are correlated with the input levels, then (3.7) is estimated as a fixed effects model and the producer specific effects are consistently estimated as

$$\hat{\eta}_{i}^{*} = \frac{-1}{T} \sum_{t} \left( \ln y_{it} - \hat{\beta}_{0}^{*} - \sum_{m=1}^{M} \hat{\beta}_{m} \ln x_{mit} - \sum_{t=2}^{T} \hat{\delta}_{t} D_{t} \right)$$
(3.9)

Alternatively, if the producer specific effects are random, then (3.7) is estimated as a random effects model<sup>5</sup> and the estimates of producer specific effects are given by-

$$\hat{\eta}_{i}^{*} = \frac{-\sigma_{\eta}^{2}}{T\sigma_{\eta}^{2} + \sigma_{\upsilon}^{2}} \sum_{t} \left( \ln y_{it} - \hat{\beta}_{0}^{*} - \sum_{m=1}^{M} \hat{\beta}_{m} \ln x_{mit} - \sum_{t=2}^{T} \hat{\delta}_{t} D_{t} \right)$$
(3.10)

The static model as represented in (3.7) omits the lagged adjustment phenomenon of inputs and is likely to provide biased estimates of technical efficiencies of the production units, particularly in the short-run, if the true process of output generation is dynamic. Also, the ranking of firms based on their technical efficiency estimates will be biased if the ranking is obtained from a similarly misspecified static model. Therefore, in the presence of sluggish adjustment of inputs, a static model cannot identify the true process of output generation or the true technical efficiency of a production system. A dynamic model is more suitable for this purpose.

## 4. Estimation Methods

The dynamic model of production as given in (3.3) includes the one period lagged dependent variable as a regressor along with other exogenous variables. Both  $y_{it}$  and  $y_{it-1}$  are functions of  $u_i$ , leading to a correlation between one of the regressors and the error term. Thus the OLS estimator is biased and inconsistent even if  $v_{it}$  are not serially correlated. Arellano and Bond (1991) suggested a generalized method of moments (GMM) estimator for the dynamic panel data model that consistently estimates a dynamic panel data model. The basic principle of such estimation is to use a first difference transformation to eliminate the

<sup>&</sup>lt;sup>5</sup> A detailed discussion on the model specification and related prediction procedures can be found in Baltagi (1995).

individual specific effects and then to consider the dependent variable with two period lags or more lags as valid instruments. The GMM estimator is more efficient than the Anderson-Hsiao (1982) instrumental variable estimator. Ahn and Schimdt (1995) derived additional non-linear moment restrictions and the estimation method is further generalized and extended by Arellano and Bover (1995) and Blundell and Bond (1998).

I use the system GMM estimator proposed by Blundell and Bond (1998)<sup>6</sup> which uses a set of moment conditions relating to the first differenced regression equation, and another set of moment conditions for the regression equation in levels. A dynamic panel data model in levels is presented by

$$y_{it} = \alpha y_{it-1} + \beta x_{it} + u_{it}, i = 1, \dots, N; t = 1, \dots, T.$$
(4.1)

where  $u_{it} = \eta_i + v_{it}$ . It is further assumed that

$$E(\eta_i) = 0, \ E(v_{it}) = 0, \ E(v_{it}\eta_i) = 0 \text{ for all } i=1,...,N \text{ and } t=2,...,T.$$
 (4.2)

$$E(v_{it}v_{is}) = 0 \text{ for all } i=1,\dots,N \text{ and } t \neq s$$

$$(4.3)$$

$$E(y_{i1}v_{it}) = 0$$
 for all  $i=1,...,N$  and  $t=2,...,T$ . (4.4)

The first difference of (4.1) does not contain the individual specific effect, and is given by -

$$\Delta y_{it} = \alpha \Delta y_{it-1} + \beta \Delta x_{it} + \Delta v_{it} \tag{4.5}$$

where  $\Delta y_{it} = y_{it} - y_{it-1}$  for i = 1, ..., N and t = 2, ..., T.

According to Bludell and Bond (1998) and Blundell, Bond, and Windmeijer (2000), the first differences of the two or more period lagged dependant variable are valid instruments for the equation in levels, and two or more period lagged dependent variables in levels are relevant instruments for the

<sup>&</sup>lt;sup>6</sup> A semiparametric estimation method for dynamic panel data models is discussed by Park, Sickles, and Simar. However, I limit the analysis of this paper within the parametric framework.

equation in first differences. In addition, some or all of the other explanatory variables  $(x_{mit})$  are exogenous or predetermined, generating more instrumental variables for estimation. For a production process, inputs are likely to be correlated with the producer specific effects and the shocks to the production system in the previous periods. Therefore, I use the following moment conditions to identify the set of valid instruments for the equations in first differences

$$E(y_{it-s}\Delta u_{it}) = 0 \text{ for } t = 3,..., T \text{ and } 2 \le s \le t-1$$
 (4.6)

$$E(x_{it-s}\Delta u_{it}) = 0 \text{ for } t = 3, ..., T \text{ and } 1 \le s \le t-1$$
 (4.7)

Further, to identify the set of instruments for the equations in levels, I use the moment conditions

$$E(u_{it}\Delta y_{it-1}) = 0 \text{ for } t = 3,..., T$$
(4.8)

and 
$$E(u_{it}\Delta x_{it-1}) = 0$$
 for  $t = 3, ..., T$  (4.9)

This GMM estimation is consistent for large N and finite T, and is more efficient that the estimator proposed by Arellano and Bond (1991).

Finally, to estimate (3.3), I use  $\ln y_{it-2}$ ,  $\ln x_{mit-1}$ , and  $\ln x_{mit-2}$ , m=1,...M, as instruments for the equation in first differences and  $(\ln y_{it-1} - \ln y_{it-2})$ ,  $(\ln x_{mit-3} - \ln x_{mit-4})$ , m=1,...,M, as instruments for the equation in levels<sup>7</sup>. I use the one-step GMM estimator for which the estimates are consistent<sup>8</sup>. A crucial assumption of the validity of GMM estimates is that the instruments are

<sup>&</sup>lt;sup>7</sup> Though it is possible to have more instrumental variables for our model, considering even deeper lags of the instrumental variables that I am using, I do not use all available instruments, as too many instruments may over fit the endogenous variable and weaken the power of the Hansen test to detect over identification. Given the sample with 28 groups, I choose to use 26 instruments from the recent lags, for which the power of the Sargan test is the largest.

<sup>&</sup>lt;sup>8</sup> While the coefficient estimates of two-step GMM estimator are asymptotically more efficient, I do not find any difference in the estimates of coefficients from the one-step and two-step estimation procedure. Since my purpose is to estimate technical efficiency, I use the one-step estimation results only.

exogenous. I verify joint validity of the instruments with the Sargan test. Furthermore, consistency of the GMM estimator relies upon the fact that the idiosyncratic errors are serially uncorrelated. If the differenced error term is second-order serially correlated, then  $\ln y_{\mu-2}$  is not a valid instrument for the first differenced equation<sup>9</sup>. The Arellano and Bond (1991) test is applied to the residuals in differences to test for second-order autocorrelation. I also employ small-sample corrections to the covariance matrix estimate, and the standard errors, which are robust to heteroskedasticity and arbitrary pattern of autocorrelation within production units.

The static model with time-invariant technical efficiency as given in equation (3.7) is estimated as a random effects model<sup>10</sup> and accordingly the technical efficiency is estimated using (3.10).

# 5. Empirical Analysis

#### 5.1. Data

To estimate the theoretical model, I use the panel data for nine years (1987/88 – 1995/96) on private sector manufacturing establishments in Egypt, obtained from the Industrial Production Statistics of the Central Agency for Public Mobilization and Statistics (CAPMAS). The data is in three-digit ISIC (International Standard Industrial Classification) level and for 28 sectors with the total number of observation being 252. The broader categories of output include

<sup>&</sup>lt;sup>9</sup> By construction, the differenced error term is expected to be first order serially correlated and the evidence of the correlation is uninformative.

<sup>&</sup>lt;sup>10</sup> Hausman's specification test (1978) for equation (3.4) using the sample suggests random effects specification.

food, tobacco, wood, paper, chemicals, non-metallic products, metallic product, engineering products, and other manufacturing products. Table 1 in the appendix presents the description of each sector.

This data set is directly taken from a study by Getachew and Sickles (2007) and details about the data can be found in their paper. They use the superlative index number approach to aggregate the data to the three-digit level, such that the establishments in each sector can be viewed as homogeneous in terms of production technology. To get a single aggregate measure of output from heterogeneous and multi-product firms, they consider total revenue from these firms for goods sold, industrial services provided to others, and so on. Finally, they obtain the quantity indices for output and inputs by deflating the total value of output and inputs by the relevant price indices.

Capital, labor, energy, and material are the inputs for the manufacturing sectors' output. As found by Getachew and Sickles (2007), the quantity indices for output and inputs grew over the period under consideration. The summary statistics of the indices are presented in Table 2 in the appendix. Getachew and Sickles (2007) use this data set to analyze relative price efficiency of the Egyptian manufacturing sectors, but they do not measure technical efficiency of these sectors, particularly, in a dynamic framework.

The private sector has always been important for the economic growth and development in Egypt. However, the Egyptian government adopted sweeping privatization policies in the early 1990 that were followed by increased growth of the private manufacturing sectors, and as a result, Egypt's manufacturing sector became the highest contributor to the value-added at the national level. Several sub-sectors of the private manufacturing sector (like food and textile) generated good employment opportunities for unskilled and semi-skilled labors, particularly in a labor abundant country like Egypt. Moreover, during the 1990s, the activities that contributed higher value-added at the national level received higher priority and as a result the input ratios were changing within different sectors (Nathan Associates Inc., 2000). I expect the production process and technical efficiency of the Egyptian private manufacturing sectors to be affected by sluggish adjustment to changing input ratios and the employment of new workers.

#### 5.2. Results

The estimation results for equation (3.3) with four inputs using the Blundell and Bond (1998) system GMM<sup>11</sup> estimator and are given in column (1) of Table 3. From the estimation results I find that the one period lagged output has a significant positive effect on the current output, where output is measured in logarithm. Using the estimated value of  $1-\hat{\lambda} = 0.16$ , the actual change in output of a sector in any period is 84% of the change in output that is needed to catch up with the potential output in that period. Further, estimate of  $(1-\hat{\lambda})$  is statistically significant at the 1% level indicating that the speed of adjustment is significantly different from unity. Assuming similar speeds of adjustment for inputs across sectors, this result supports the partial adjustment scheme for output and suggests that the static model is a misspecified one for this sample.

As the purpose of this paper is to identify the true technical efficiency of the sectors, the significance levels of the input elasticities are not of much interest.

<sup>&</sup>lt;sup>11</sup> I use Stata command xtabond2 developed by Roodman (2006). The standard errors of the estimates are robust to heteroskedasticity and arbitrary patterns of autocorrelation within sectors, and I also incorporate the small-sample corrections to the covariance matrix estimate.

However, I find that labor and material have significant input elasticities in the dynamic production model<sup>12</sup>.

Consistency of the system GMM estimator relies upon the fact that the idiosyncratic errors are not serially correlated. The AR(2) test statistic (p-value = 0.966), as reported in column (1) of Table 3 corresponds to the test of the null hypothesis that the residuals in the first-differenced regression exhibit no second order serial correlation. Following the test procedure proposed by Arellano and Bond (1991), a negative first order serial correlation in the equation in first differences is expected and the AR(1) test statistic supports that. Thus, the random shocks to the sectors are not serially correlated and the estimation results are consistent.

The Sargan test statistic for testing exogeneity of the instrumental variables, as reported in column (1) of Table 3, supports validity of the instruments (p-value = 0.688). The GMM system estimation uses internal instruments for estimation, and thus, there can be several valid instrumental variables. I chose the set of instrumental variables for which the Sargan test of exogeneity was the most powerful.

To generate the comparative estimates of technical efficiency, I estimate (3.7), the static stochastic frontier, as a random effects model, following the Hausman's specification test (1978) results. The estimation results presented in column (2) of Table 3 show that the input elasticities are not materially different from those estimated using the dynamic model<sup>13</sup>. The estimation results,

<sup>&</sup>lt;sup>12</sup> Though capital and energy do not have significant input elasticities, I do not drop them from our production model, because they are valid inputs, and have positive elasticities as expected.

<sup>&</sup>lt;sup>13</sup> The estimation results from a fixed effects model are similar to the random effects model. The input elasticities of capital, labor, energy, and material are 0.15, 0.92, 0.014, and 0.76 respectively when (3.6) is estimated as a fixed effects model. The elasticity of material is significant at the 1% level among other inputs.

as presented in Table 3, also show an interesting phenomenon regarding the input elasticities. The coefficients of the input variables, as estimated from the dynamic model represent the short-run input elasticities. The long-run input elasticities are obtained by dividing the estimated coefficients by  $\hat{\lambda} = 0.84$  for the data used in this paper, and are presented in column (3) of Table 3. Thus, the long-run input elasticities are higher than the short-run elasticities. This is due to the fact that, in the presence of sluggish adjustment of inputs in the short-run, the inputs cannot contribute at their full capacity, and hence, their contribution to the change in output in the short-run is less than what the inputs can contribute in the long-run. However, I cannot really compare the estimates from the dynamic and the static model as the short-run and long-run elasticities because, for this sample, the static model is misspecified and yields biased estimates of parameters.

The average of the time-invariant technical efficiency estimates from the dynamic (3.3) and the static (3.7) models are shown in column (1) and (2) of Table 4. I find that the technical efficiency estimate from the dynamic model is 74.5% for a sector on average, whereas the estimate from the static model is 70.02% on average. Thus, I find that the static model underestimates the technical efficiency of sectors by 4.49 percentage points on average<sup>14</sup>, and this underestimation can be as high as 17.16 percentage points. Nine sectors or almost one-third of the total, have differentials as large as 10 percentage points.

The estimates of technical efficiency from the dynamic and the static specification of production model clearly suggest that the static model generates biased estimates of technical efficiency in the presence of lagged adjustment of inputs. Due to the fact that only relative efficiency has been measured using the

<sup>&</sup>lt;sup>14</sup> Average TE from dynamic model – Average TE from static model = 74.51 - 70.02 = 4.49 percentage points.

stochastic frontier approach, the technical efficiency estimates from the static model can be either higher or lower than the estimates from the dynamic model for a particular sector. It is important to note that in nineteen of the twenty eight sectors technical efficiency is underestimated by the static model, and in nine of these by as much as 10 percentage points.

Wilcoxon signed-rank test to check whether the median difference of the technical efficiency scores from the dynamic and the static model is different from zero also shows that the technical efficiency estimates from the dynamic and the static model are significantly different. More specifically, p-value while testing the null hypothesis that difference between estimated efficiency scores from the dynamic and the static model has median zero is 0.0091. Consequently, I reject the null hypothesis.<sup>15</sup>

The misspecification of the static model also causes the ranking of sectors according to the dynamic and static model specifications to differ. The ranking of sectors based on their technical efficiency estimates are given in column (3) and (4) of Table 4. The two model specifications, though agreeing on the best sector over all, generate different internal ranking for the other sectors. The Spearman's correlation coefficient for these ranks from the dynamic and static model is found to be 0.59. Though the ranks of sectors as generated from the static and the dynamic model may not be independent, clearly they are different. Since several organizational and production decisions are taken based on the relative efficiency

<sup>15</sup> The Wilcoxon signed-rank test statistic is given by  $z = \left\{\frac{1}{4}n(n+1) - T - \frac{1}{2}\right\} / \left\{\sqrt{\frac{1}{24}n(n+1)(2n+1)}\right\}$ where *n* is the number of non-zero differenced terms, *P* is the positive signed ranks, and  $T = \frac{1}{2}n(n+1) - P$ . of the sectors, a true ranking as generated by the dynamic model is more reliable<sup>16</sup>.

## 6. Conclusion

This paper outlines a theory for a dynamic stochastic production frontier that describes a process of lagged output response to sluggish adjustment of inputs, and accordingly measures the time-invariant technical efficiency of production units using a dynamic production model. Using data from the private manufacturing sectors in Egypt, I find that the speed of adjustment of output is significantly lower than unity for the period under consideration. Thus, the conventional static model that assumes instantaneous adjustment of inputs is missspecified, and provides biased estimates of technical efficiency.

The dynamic production model provides a more realistic approach to estimating technical efficiency in Egyptian manufacturing, where sluggish adjustment of inputs is a very plausible phenomenon in light of the fact that during the period under consideration, Egypt underwent several changes in the manufacturing production. Our analysis shows 19 of 28 sectors were relatively more efficient than static model estimates would reveal.

Estimation of technical efficiency and ranking of production units according to their efficiency levels are important aspects of productivity analysis, forming the basis for critical decisions about their production plans and informing policy makers about the relative performances of the production units. For example technical efficiency estimates can identify whether publicly owned or

<sup>&</sup>lt;sup>16</sup> The estimated coefficients of the time dummies, both from the static and the dynamic model, were negative through 1991/92, and positive thereafter. the effects of technological changes on output were negative. However, with a significant effect in 1990/91 only, the pattern of the coefficients support the positive impacts on the Egyptian industrial output of the introduction of the new economic reform programs in early 1990s.

privately owned companies are more efficient, or whether there is any change in efficiency after a policy intervention. Therefore, better estimates of technical efficiency will result better decisions, and on average, better outcomes.

# **Appendix: Tables and Figures**

# Table 1: Sectors and the Industrial Activities at the three-digit ISIClevel

Sector Number	Industrial activity		
1	Food manufacturing		
2	Other food manufacturing		
3	Beverage and liquor		
4	Tobacco		
5	Manufacture of textile		
6	Manufacture of wearing apparels		
7	Manufacture of leather products		
8	Manufacture of footwear		
9	Manufacture of wood products		
10	Manufacture of furniture & fixture		
11	Manufacture of paper products		
12	Printing and publishing industries		
13	Manufacture of industrial chemicals		
14	Manufacture of other chemical products		
15	Other petroleum and coal		
16	Manufacture of rubber products		
17	Manufacture of plastic products		
18	Manufacture of pottery and china		
19	Manufacture of glass and glass products		
20	Manufacture of other non metallic products		
21	Iron and steel basic industries		
22	Non-ferrous basic industries		
23	Manufacture of fabricated metal products		
24	Manufacture of machinery except electrical		
25	Manufacture of electrical machinery		
26	Manufacture of transport equipment		
27	Manufacture of professional equipment		
28	Other manufacture industries		

Table 2: Variable Descriptions and	Summary Statistics
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Variable	Description	Number of Observation	Mean	Standard deviation	Minimum	Maximum
Yearid	id number for 9 years of data for each sector	252	5	2.59	1	9
Sectorid	id numbers for the 28 three digit manufacturing sectors	252	14.5	8.09	1	28
Output	Output quantity index	252	2888.90	3333.39	67	19236
Capital	Capital quantity index	252	288.84	475.29	1	3437
Labor	Labor quantity index	252	273.34	344.06	10.50	1689.2
Energy	Energy quantity index	252	61.97	116.56	0.20	860.1
Material	Material quantity index	252	1823.44	2168.83	44.8	11853.8

Source: Getachew and Sickles (2007).

# Table 3: Estimation Results from Dynamic and Static Specifications(Time-Invariant Technical Efficiency Model)

	Dynamic Specification	Static Specification	Long-run Input Elasticity
Coefficients	(1)	(2)	(3)
	0.16***		
lag_In(output)	[0.06]	-	-
	0.02	0.014	0.02
In(capital)	[0.05]	[0.01]	
	0.22**	0.123***	0.26
In(labor)	[0.09]	[0.04]	
	0.04	0.044**	0.05
In(energy)	[0.05]	[0.02]	
	0.65***	0.833***	0.77
In(material)	[0.09]	[0.03]	
	0.33	0.803***	
Constant	[0.29]	[0.15]	-
AR(1)	-2.72***	-	-
AR(2)	-0.04	-	-
Sargan test	12.79	-	-
Observations	140	252	-
Number of sectors	28	28	-
Number of instruments	26	-	-
R-squared	-	0.973	-
F (8, 27)	381.21***	-	-

*Note*: Robust standard errors are reported in parentheses; \* significant at 10%; \*\* significant at 5%; \*\*\* significant at 1%; AR(1) and AR(2) represent the Arellano-Bond (1991) test statistics for the first order and second order serial correlation in the first differenced residuals respectively; The null hypothesis for Sargan test is that instruments used are not correlated with the residuals; the instruments used are  $ln(output)_{it-2}$ ,  $ln(capital)_{it-1}$ ,  $ln(capital)_{it-2}$ ,  $ln(labor)_{it-1}$ ,  $ln(labor)_{it-2}$ ,  $ln(energy)_{it-1}$ ,  $ln(energy)_{it-2}$ ,  $ln(material)_{it-3} - ln(capital)_{it-2}$  for the equation in first differences, and are  $ln(output)_{it-2}$ ,  $ln(capital)_{it-3} - ln(capital)_{it-3} - ln(labor)_{it-4}$ ,  $ln(energy)_{it-3} - ln(energy)_{it-4}$ , and  $ln(material)_{it-3} - ln(material)_{it-4}$  for the equation in levels; The regressions also include dummy variables for the different time periods that are not reported.

# Table 4: Time-Invariant Technical Efficiency Estimates and Ranking ofSectors from Static and Dynamic Specifications

Sectorid	Technical Efficiency from Dynamic Specification (%)	Technical Efficiency from Static Specification (%)	Underestimation by Static Model	Rank_Dynamic Specification	Rank_Static Specification
	(1)	(2)	(1) - (2)	(3)	(4)
1	57.12	64.35	-7.23	27	21
2	79.62	70.94	8.68	6	11
3	68.91	66.88	2.03	21	17
4	87.51	74.28	13.23	4	6
5	54.17	61.84	-7.67	28	25
6	65.81	65.69	0.12	23	20
7	71.46	59.56	11.9	17	28
8	71.27	65.87	5.4	19	19
9	78.46	66.34	12.12	9	18
10	76.63	73.58	3.05	11	8
11	100	100	0	1	1
12	64.58	68.5	-3.92	24	14
13	76.37	67.9	8.47	13	16
14	73.13	71.96	1.17	15	9
15	78.58	61.42	17.16	8	26
16	76.2	62.66	13.54	14	24
17	60.64	64.3	-3.66	26	22
18	77.11	71.78	5.33	10	10
19	76.59	77.72	-1.13	12	4
20	71.41	78.14	-6.73	18	3
21	99.67	86.2	13.47	2	2
22	72.54	60.19	12.35	16	27
23	65.86	70.23	-4.37	22	12
24	60.81	63.29	-2.48	25	23
25	69.69	68.1	1.59	20	15
26	88.96	76.2	12.76	3	5
27	78.76	73.8	4.96	7	7
28	84.29	68.81	15.48	5	13
Mean	74.51	70.02	4.49	_	
Maximum	100.00	100.00	17.16	_	_

*Note*: Technical efficiency of a sector is measured relative to the most efficient sector.

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